1 Introduction

In this paper, we explore the extent to which capacity regulation may be used as a means to regulate the quality provision in a deregulated industry. To this end we consider a vertically differentiated industry where the government regulates the installed production capacity of an incumbent before an entrant selects its product quality and compete in prices afterwards. We establish two main results. First, we offer an original characterization of a firm's equilibrium payoff under a particular class of Bertrand-Edgeworth pricing games with product differentiation. Second, we compare the effects of capacity regulation with those of MQS, when set at their industry welfare maximizing values and show that capacity regulation systematically dominates the MQS instrument.

The extent to which governments may improve quality provision in oligopolistic markets has been studied for long. Whenever consumers exhibit different willingness to pay for quality, firms are inclined to relax price competition by offering products of differing qualities. There is then a general presumption that the industry quality supply is insufficient from a social point of view. If the quality dimension of those products pertains to safety, health, or any other attribute that might alter consumers' quality of life, there is a case for direct regulation of quality. This is especially true if quality is not immediately verifiable by the end-users. Governments should then edict, and enforce, minimum requirements regarding safety, reliability or health attributes in order that products be allowed to compete in the market place. In many instances though, whenever products satisfy these minimum requirements, their quality is perfectly observable and/or whenever we focus our attention on those other dimensions of quality that do not endanger consumers' integrity, quality under-provision is still a relevant policy issue because it is a form of market failure that results from imperfect competition.

Consider for instance broadband access to the Internet. Depending on their income or professional activities, different consumers may be willing to pay different prices for different connection speeds or download volumes. Offering different quality-price menus is an obvious strategy aimed at segmenting the market (and increase profits). However, because of the network externalities associated to broadband access, or more simply to avoid the so-called digital divide, governments may wish to implement a policy that imposes minimal speed standards to access providers. Comparable arguments could as well be developed regarding environmental friendly technologies, or products: even though consumers positively value environmental compliance, they do it with various intensities. Firms' willingness to segment market will very likely result into a range of products displaying too little environmental friendliness.

The literature typically assumes that governments rely on Minimum Quality Standards (hereafter MQS) to ensure quality upgrades, well beyond safety or public health standards. The seminal paper in this literature is Ronnen (1991). It shows that the adequate selection of a MQS can increase both quality and sales so that the industry welfare unambiguously increases. The intuition for this positive result is quite simple: by constraining the low quality firm to upgrade its quality, the MQS induces the high quality firm to select a still higher quality (in order to relax competition). In equilibrium, the price competition is however fiercer so that prices are lower and more consumers end up participating. Crampes and Hollander (1995) establish a qualitatively similar result with a different costs structure.

These two papers obviously make a case for MQS but their conclusions might be challenged on several grounds. Firstly, Ronnen (1991)'s results in favor of imposing a MQS have formal validity only in a neighborhood of the unregulated level (cf. his theorem 5). Next comes the issue of certification that inevitably goes along with MQS.[1] In this respect, Albano and Lizzeri (2001) show that certification does not go without inefficiencies: although certification intermediaries tend to raise firms' incentives to provide quality, they are likely to fail in avoiding quality under-provision. Finally, the MQS instrument itself exhibits several drawbacks. Valletti (2000) shows that Ronnen (1991)'s mechanism is not robust to the mode

[1]Regarding informational issues raised by quality provision in deregulated markets, we refer the reader to Auriol (1998).

of competition: the switch from Bertrand to Cournot type competition destroys the "good" incentives to increase qualities. On a different tack, Scarpa (1998) shows that the welfare enhancing effect might critically depend on the duopolistic structure of the industry. Maxwell (1998) then puts MQS in a dynamic perspective and shows that they decrease welfare in the long run because they weaken incentives to innovate. Lutz, Lyon, and Maxwell (2000) provide a model where firms may manipulate the selection of the MQS by the regulator in such a way that industry welfare actually decreases. Glass (2001) reaches similar conclusions in a slightly different setup. Interestingly enough, these cases against MQS are rooted in its most obvious implication: *a MQS undermines industry's profitability*. As a by-product, imposing a MQS might induce the exit of some firms, or reduce entry, a problem also acknowledged in Ronnen (1991) and Crampes and Hollander (1995).[2]

This exit issue is quite problematic for those industries which are presently subject to deregulation. Recurrent incidents in the US electricity market (2001 California crisis, 2003 black-out) or UK railways over the last decades (cf. Reuters news) suggest that quality might indeed be a concern during the deregulation process.[3] Anecdotal evidence from the broadband internet access also suggests that at the early stage, entrants tend to challenge the incumbent, most often the former monopoly which controls the telecommunication network, by offering lower prices for services which turn out to be of a lower quality (longer connection delays, limited reliability, limited technical support).

The virtues of MQS as a means to ensure quality provision are thus somewhat mixed. This is why we propose to explore an alternative theoretical route, namely that of capacity regulation. While the original characterization of a firm's equilibrium payoff under a particular class of Bertrand-Edgeworth pricing games is essentially a theoretical result, we also consider normative issues. In particular, we

[2]Notice that this mixed theoretical appraisal of MQS is to some extent confirmed by the (limited) empirical evidence. See in particular Chipty and Witte (1997) for a detailed empirical study of the effects of MQS on the quality of child care centers in the US.

[3]Evidences of the negative effect of deregulation in US Airline markets on the service quality can also be found in Rhoades and Waguespack (2000).

compare the effects of capacity regulation with those of MQS, when set at their industry welfare maximizing values and show that capacity regulation systematically dominates the MQS instrument. The intuition underlying our result is easy to summarize. On the one hand, it has been shown for long (cf. Kreps and Scheinkman (1983)) that the presence of capacity limitations relaxes price competition. On the other hand, whenever firms strategically downgrade products' quality, this is motivated by their willingness to relax price competition as well. Thus, if some form of capacity limitation is introduced in the price competition game, firms may be less inclined to downgrade quality, since competition is, somehow, already relaxed. We develop a stylized duopoly stage game in which this intuition is proved to be correct. More precisely, we show that by introducing capacity regulation, a regulator may completely the entrant's incentives at the quality selection stage: there is no strategic reason left for downgrading quality and the two firms end up offering the best available, efficient, quality.

Even though the intuition underlying our result looks straightforward, it must be noted that the formal proof is actually quite challenging since we have to deal with Bertrand-Edgeworth price competition under product differentiation. Very little is known indeed regarding the equilibrium structure for this class of pricing games.[4] Krishna (1989), Furth and Kovenock (1993) and Cabral, Kujal, and Petrakis (1998) offer some partial characterizations of Nash equilibrium in prices for the case of horizontal differentiation but do not study their implication on product differentiation. In Boccard and Wauthy (2006), we offer preliminary results for the case of vertical differentiation. In that paper, we already showed that quantity regulation imposed on an incumbent firm may be helpful in ensuring quality provision. However the analysis reported there is only a local one. More precisely it is confined to particular exogenous levels of the quantitative restriction.[5] Since the

[4]Strictly speaking, most of the results established in the oligopoly pricing games literature under product differentiation are simply not robust to the introduction of decreasing returns to scale.

[5]Notice also that Boccard and Wauthy (2006) essentially focus on the role of the competition mode in assessing the effects of quantitative constraints whereas the present paper systematically focuses on the comparison between MQS and quantitative restrictions under price competition.

quantitative restriction is endogenous in the present paper, we need to develop a more general framework in order to characterize firms payoffs in the pricing game globally, i.e. whatever the value of the quantitative restriction. While being technical in nature, these original results should prove useful to develop a more thorough analysis of Bertrand-Edgeworth competition under product differentiation in general settings.

On the normative side, our results are admittedly extreme: capacity regulation always welfare dominates MQS through the effect it has on quality selection; under capacity regulation, equilibrium quality levels are simply the efficient ones. Of course, such results should not be taken literally. MQS definitely remain necessary as a certification device when quality is not perfectly observable by consumers and/or as a means of ensuring minimal safety requirements. However, our analysis suggests that as a mean of controlling for firms' strategic incentives to downgrade quality (within the admissible range), MQS are a relatively ineffective instrument, as compared to capacity regulation. We view our analysis as particularly relevant to the analysis of industries subject to deregulation. In these industries, the incumbent is clearly identified, and plausibly subject to some residual regulation. Although it is not that pervasive as a regulation policy, limiting the production capacity of the incumbent seems quite natural as a tool to invite entry as it ensures the entrant a protected (though limited) market share. The current regulation framework of various European industries allows for such a regulation. A concrete example is the italian electricity market where a new law prohibits any generation company from supplying more than one half of the national demand. This measure was successfully taken to induce entry of competitors to challenge the historical incumbent. A comparable provision can be found in the European Regulation on Deregulation of Public Transport whereby the regulator may choose to limit market coverage of an already dominant firm in order to allow for enough competition. More precisely, Article 9 states that *"A competent authority may decide not to award public services contracts to any operator that already has or would, as a consequence, have* *more than a quarter of the value of the relevant market...”*.[6] In the present paper, we show that such policies also display very nice complementarities regarding the regulation of quality provision.

2 Premises

We consider a regulator R, an incumbent firm i and a potential entrant firm e interacting in the stage game Γ depicted in Figure 1. Our equilibrium concept is subgame perfect equilibrium (SPE). The regulator may adopt a “Laissez-Faire” (LF) attitude or be more active with either the enforcement of a minimum quality standard (MQS) or the imposition of a sales, or capacity, restriction (SR) (over the incumbent). Each possible strategy gives rise to a subgame where the entrant has to decide whether to enter, and if so, pick a quality level before engaging into price competition with the incumbent.

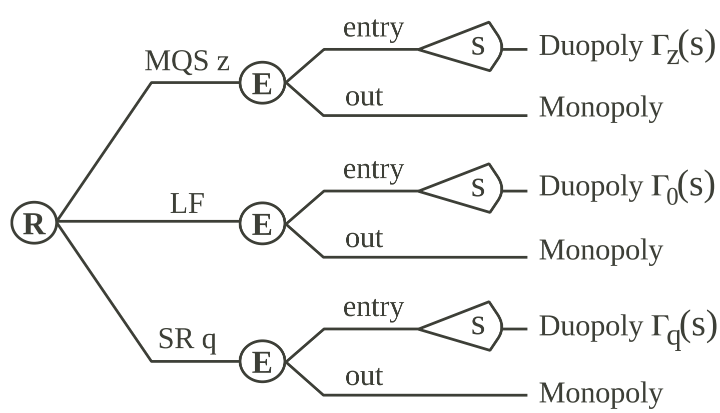

Figure 1: The stage game Γ

[6] Amended proposal for a regulation of the European Parliament and of the council on action by member states concerning public service requirements and the award of public sector contracts in passenger transport by rail, road and inland waterway, Official Journal of the European Commission, C 151 E/146-183, Article 9-2.

Section 2.1 presents the details of the model and solves the Laissez-Faire case. Section 2.2 characterizes the equilibrium under a minimum quality standard and derives the preferred MQS of the regulator. Section 3 then brings the necessary modifications to study price competition under a binding capacity limit. We derive the optimal quality choice of the entrant and his optimal entry strategy conditional on the governmental capacity restriction. After characterizing the preferred capacity restriction for the regulator in section 3.5, we can in section 3.6 compare the three instruments. Section 4 concludes.

2.1 "Laissez-Faire"

The following hypothesis apply for the entire game Γ. In order to better focus on the relative merits of MQS and CR as regulatory instruments, we assume that quality is not costly for firms and that the marginal cost of production is nil.[7] Secondly, we assume that the incumbent i is committed to the best available quality (normalized to unity)[8] so that the entrant e cannot leapfrog him. In formulas, we set $s_i = 1, F_i = 0, s_e = s \in [0,1], F_e = F > 0$.

Like the literature on MQS, we follow Mussa and Rosen (1978) and (Tirole, 1988, sec. 2.1) to model quality differentiation. A consumer with personal characteristic x is willing to pay xs for one unit of quality s and nothing more for additional units. He maximizes surplus and when indifferent between two products, selects his purchase randomly. Types are uniformly distributed in $[0,1]$. The indirect utility function of a consumer with type x buying a product with quality s at price p is therefore defined as

$$U(x,s) = xs - p$$

[7] These extreme assumptions allow us to focus exclusively on the strategic incentives to differentiate by quality that are aimed at relaxing price competition.

[8] An upper bound on the admissible qualities is required to ensure that firms' payoffs are bounded. We performed the analysis with the cost of quality $k(s) = \frac{s^2}{K}$ for $s \in [0,1]$ without notably affecting the qualitative conclusions of our analysis. The computations can be found in Boccard and Wauthy (1998). Notice that our cost assumption amounts to choose K arbitrarily large.

Under "Laissez-Faire" (subgame Γ_0), the challenger decides whether to enter or not, and if so, chooses her quality s and pays a sunk cost $F \geq 0$. In the second stage, denoted $\Gamma_0(s)$, the two firms sell goods differentiated by their quality and compete in prices. We study the Subgame Perfect Equilibria of Γ_0.

We may now characterize demands addressed to the firms. Whenever $s = 1$, any consumer is indifferent between the two products whenever they are sold at the same price; if they are sold at different prices, any consumer prefers the cheapest product. Demand addressed to firm i is therefore discontinuous, as is usual under Bertrand competition with homogeneous products. Under our assumptions about consumers' preferences, the formal definition of $D_i(\cdot)$ is as follows:

$$D_i(p_i, p_j) = \begin{cases} 1 - p_i & \text{if} \quad p_i < p_j \\ \frac{1-p_i}{2} & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j. \end{cases} \tag{1}$$

Whenever $s < 1$, i.e. whenever products are differentiated, we identify the indifferent consumer, $\tilde{x}(p_i, p_e)$ who, by definition, satisfies equation $x - p_i = sx - p_e$, as well as the marginal consumer $\overline{x}(p_e)$ who, by definition, is indifferent between buying product e and refraining from consuming any of the good. He solves equation $sx - p_e = 0$. Formally we find

$$\tilde{x}(p_i, p_e) = \frac{p_i - p_e}{1 - s} \text{ and } \overline{x}(p_e) = \frac{p_e}{s}$$

Demand addressed to the incumbent is defined by the set of consumers $x \in [\tilde{x}(\cdot), 1]$ and demand addressed to firm e is defined by the interval of types $x \in [\overline{x}(\cdot), \tilde{x}(\cdot)]$. Taking non-negativity constraints into account, we formally define the following piecewise linear demand functions:

$$D_e(p_e, p_i) = \begin{cases} 1 - \frac{p_e}{s} & \text{if} \quad p_e \leq p_i - 1 + s \\ \frac{p_i s - p_e}{s(1-s)} & \text{if} \quad p_i - 1 + s \leq p_e \leq p_i s \\ 0 & \text{if} \quad p_e \geq p_i s \end{cases} \quad (2)$$

$$D_i(p_e, p_i) = \begin{cases} 1 - \frac{p_i}{1} & \text{if} \quad p_i \leq \frac{p_e}{s} \\ 1 - \frac{p_i - p_e}{1-s} & \text{if} \quad \frac{p_e}{s} \leq p_i \leq p_e + 1 - s \\ 0 & \text{if} \quad p_i \geq p_e + 1 - s. \end{cases} \quad (3)$$

The characterization of Nash equilibria in the pricing game $\Gamma_0(s)$ is done in Choi and Shin (1992). A full-fledged analysis of this equilibrium is available in the Appendix. It is indeed essentially a matter of computations to derive firms' best reply functions. There are piecewise linear and continuous and allow the characterize the unique Nash equilibrium that follows:

$$p_e^* = \frac{s(1-s)}{4-s} \quad \text{and} \quad p_i^* = \frac{2(1-s)}{4-s} \quad (4)$$

Plugging (12) into (9), we obtain the first stage payoffs as a function of the entrant's quality

$$\Pi_i = \frac{4\,(1-s)}{(4-s)^2} \quad \text{and} \quad \Pi_e = \frac{s\,(1-s)}{(4-s)^2} \quad (5)$$

We can now solve for the subgame perfect equilibrium (in pure strategies) of the "Laissez-Faire" game Γ_0. The entrant's optimal choice is the solution $s = \frac{4}{7}$ to the FOC $\frac{\partial \Pi_e}{\partial s} = 0$; it leads to a payoff of $\frac{1}{48}$ in equilibrium. Accordingly, entry will take place if only $F \leq \frac{1}{48}$. The "Laissez-Faire" analysis is summarized in the next Lemma.

Lemma 1 *Suppose quality is not costly and the incumbent sells quality $s_i = 1$, then whenever $F \leq \frac{1}{48}$, the entrant enters and optimally differentiates by selecting quality $s = \frac{4}{7}$. The price equilibrium of the continuation game is unique and in pure strategies.*

For later use, we notice that equilibrium sales are $D_i^* = \frac{7}{12}$ and $D_e^* = \frac{7}{24}$ i.e., the incumbent sells twice as much as the entrant.

2.2 Minimum Quality Standard

In this section, the government commits to a minimum quality standard (MQS) $0 \leq z \leq 1$ (since the MQS cannot exceed the best feasible quality). The continuation game played by the two firms is denoted Γ_z.

Consider the case where the challenger has entered the market. Obviously, a MQS lower than the Laissez-Faire equilibrium level $\frac{4}{7}$ leaves the entrant unbothered whereas a higher level leads him to stick to the lowest admissible quality level z. Relying on our previous analysis, we replace s by z in equations (12). The resulting price equilibrium is

$$p_e^z = \frac{z(1-z)}{4-z} \quad \text{and} \quad p_i^z = \frac{2(1-z)}{4-z},$$

leading to demands $D_i^z = 2$, $D_e^z = \frac{2}{4-z}$. Equilibrium profits are:

$$\Pi_e^z = \frac{z\,(1-z)}{(4-z)^2} \quad \text{and} \quad \Pi_i^z = \frac{4\,(1-z)}{(4-z)^2}.$$

In order to characterize the optimal MQS for the regulator, we assume that her objective is to maximize market welfare and that the enforcement costs of a MQS are nil. Net of the sunk entry cost F, the industry welfare is defined by the following expression:

$$\underline{W}(z) = \int_{1-D_i^z}^{1} (x - p_i^z) dx + \int_{1-D_e^z-D_i^z}^{1-D_i^z} (zx - p_e^z)\, dx + \Pi_i^z + \Pi_e^z = \frac{12 - z - 2z^2}{2\,(4-z)^2} \qquad (6)$$

where the first two terms denote the surplus of consumers buying the high and low quality product respectively. This function is increasing and concave in z.[9] Notice that $\underline{W}(z)$ ranges from $\frac{3}{8}$ to $\frac{1}{2}$ over the range $[0,1]$.

Incidentally, $\underline{W}(1) = \frac{1}{2}$ also defines the first best for this industry, when there are no entry sunk cost. This corresponds to the case where all consumers buy the best available quality at marginal cost (which is zero in the present case).

[9] We have $\underline{W}''(z) = -\frac{17z+4}{(4-z)^4} < 0 < \underline{W}'(z) = \frac{20-17z}{2(4-z)^3}$.

This outcome would be achieved if there were two firms in the market, competing in price with an homogeneous product of quality $s = 1$. Since firms derive no economic rent in this equilibrium, entry would take place if only the fixed cost for quality F is nil i.e., the long term "free entry" hypothesis of perfect competition holds.

Whenever the entry cost is strictly positive, the regulator must distinguish whether entry occurs or not as a consequence of her choice of the MQS z. In the absence of entry, the top quality monopoly incumbent serves half of the market at the monopoly price $p^M = \frac{1}{2}$ and generates a welfare of $\frac{3}{8}$, incidentally equal to $\underline{W}(0)$. In case of entry, welfare is $\underline{W}(z)$ and since this is an increasing function, the preferred choice of the regulator is the largest MQS compatible with entry.

There is thus a tension between the presence of a MQS and entry because the higher the MQS, the closer the two versions of the product, the tougher the price competition and the lower the entrant's profits. Since it is necessary to recoup the entry sunk cost, its level determines the maximum MQS that can be successfully implemented, i.e. that is compatible with entry. Formally, this argument summarizes as follows:

Proposition 1 *Whatever the fixed cost F in $\left[0, \frac{1}{48}\right]$, there exists a MQS $z^*(F)$ that maximizes industry welfare subject to the entry of the challenger. Both $z^*(F)$ and $\underline{W}(z^*(F))$ decrease with F.*

Proof: The upper bound for the MQS is given by the level $z^*(F)$ for which an entrant's profit, net of the entry cost is zero. Solving for $\Pi_e^z = F$, we obtain $z^*(F) = \frac{1+8F+\sqrt{1-48F}}{2(1+F)}$, as the unique relevant root (with $z^*(0) = 1$ as expected). As $z^{*\prime}(F) = \frac{24F+7\sqrt{1-48F}-25}{2\sqrt{1-48F}(1+F)^2} < 0 \Leftrightarrow 576(1+F)^2 > 0$, this function is decreasing over the domain $F \in [0, \frac{1}{48}]$ and since $\underline{W}(.)$ is increasing, total welfare is a decreasing function of the sunk cost over $[0, \frac{1}{48}]$. ■

Corollary 1 *When F increases from 0 to $\frac{1}{48}$, the duopoly regime prevails; the net surplus $\underline{W}(z^*(F)) - F$ is concave decreasing with limit $\frac{7}{16} \simeq 0.437$. For $F > \frac{1}{48}$, the monopoly regime prevails and welfare drops to $\frac{3}{8} = 0.375$.*

3 Capacity Regulation

In this section, we assume that the regulator imposes a capacity restriction q upon the incumbent. According to this regulation, the sales of firm i cannot exceed q so that, from the firm's point of view, this restriction is equivalent to the presence of an exogenous capacity constraint. We analyze now the ensuing pricing game Γ_q played by the firms.

3.1 Price Competition with a Capacity Restriction

By definition, the sales quota q defines the largest demand level the incumbent is allowed to serve. This restriction deeply alters the nature of competition in the pricing game $\Gamma_q(s)$. Indeed, whenever prices are such that the demand $D_i(p_{i,}p_e)$ is greater than q, the incumbent must turn $D_i(p_{i,}p_e) - q$ consumers away in order to comply with the capacity restriction. In other words, the incumbent must ration consumers when demand addressed to him exceeds the quota. The key implication of the capacity restriction is thus to induce Bertrand-Edgeworth competition at the pricing stage of the game. As is well-known, the organization of rationing in the market is a critical issue for such games.[10] We assume that the *efficient rationing rule* is at work i.e., whenever $D_i(p_i, p_e) > q$, rationed consumers are those who exhibit the lowest willingness to pay for the good.

We now turn to the analysis of the pricing subgames. Two classes of Bertrand-Edgeworth pricing games have to be distinguished according to the quality selected in the first stage:

- If $s = 1$, firms sell homogeneous products in the price game and one of them faces a quantitative constraint. We shall refer to Levitan and Shubik (1972) for a detailed analysis of the price equilibrium in these subgames.

[10] See Davidson and Deneckere (1986) for a classical analysis of this last issue.

- If $s < 1$, we have a Bertrand-Edgeworth pricing game with product differentiation. We present hereafter an original characterization of equilibrium payoffs for such games.[11]

We start by analyzing subgames where products are differentiated ($s < 1$) and then turn to the case of homogeneous products before concluding with the optimal quality choice by the entrant.

3.2 Differentiated Products

In order to study the pricing game $\Gamma_q(s)$ for $s_e = s < 1 = s_i$, we may divide the price space into a binding and a competitive regime according to whether the sales constraint is active or not. Under efficient rationing, it is easy to show that when a consumer wishes to buy the high quality product but is rationed by the incumbent, he always prefers to buy the low quality product of the entrant rather than not consuming. Thus, when at the prevailing prices, the demand addressed to the incumbent exceeds the quota q, all rationed consumers are recovered by the entrant, who faces a residual market $1-q$.

When rationing is at work, demands addressed to the firms typically differ from their effective sales. Using the demand equation (3), we derive the solution of $D_i(p_e, p_i) = q$ as $p_i = \beta^{-1}(p_e) \equiv \min\{1-q, p_e + (1-q)(1-s)\}$ which is a non decreasing function. The sales of both firms are therefore

$$S_i(p_e, p_i) = \begin{cases} D_i(p_e, p_i) & \text{if} \quad p_e \leq \beta(p_i) \\ q & \text{if} \quad p_e \geq \beta(p_i) \end{cases} \tag{7}$$

and

$$S_e(p_e, p_i) = \begin{cases} D_e(p_e, p_i) & \text{if} \quad p_e \leq \beta(p_i) \\ 1-q-\frac{p_e}{s} & \text{if} \quad p_e \geq \beta(p_i) \end{cases} \tag{8}$$

[11] Furth and Kovenock (1993) provide some characterization of equilibrium payoffs in Bertrand-Edgeworth games of horizontal product differentiation with sequential pricing decisions. Boccard and Wauthy (2006) provide a partial characterization for the case of vertical differentiation.

The first branch in each sales' equation applies in the competitive regime; the second branch applies in the binding regime. Notice, as a preliminary observation, that within the binding domain, the incumbent's sales are constant so that the optimal price is simply the highest price for which the quota is binding. As for the entrant, he holds a monopoly over a protected market of size $1-q$ so that his optimal price is also independent of the incumbent's one. The key-point then is to note that the possibility of rationing breaks the concavity of the entrant's profit function whereas that of the incumbent's remains concave but only over the domain where his demand is positive (recall from (3) that D_i becomes nil for $p_i > p_e + 1 - s$).

This phenomenon will preclude the existence of pure strategy equilibria in many pricing subgames. While the existence of mixed strategy equilibria is not an issue here because payoffs are continuous, the characterization of mixed strategy equilibria in Bertrand-Edgeworth games with product differentiation is to a large extent an open problem. To the best of our knowledge, Krishna (1989) provides the first characterization of a mixed strategy equilibrium in a model of symmetric product differentiation. The structure of the mixed strategy equilibrium she identifies can be used within our setup. It takes the following form: the entrant will mix over two atoms (the security price and some lower price) while the incumbent will play a pure strategy. However, in many subgames, this equilibrium does not exist because a crucial non-negativity constraint is not satisfied for the incumbent. While we do not characterize equilibria explicitly, we are able to characterize the entrant's equilibrium payoff for such cases.

The following proposition constitutes the main technical contribution of this article to the literature on Bertrand-Edgeworth competition with product differentiation.

Proposition 2 *For $s < 1$, there exists a critical value $\bar{q}(s)$ for the quota such that*
- *if $q > \bar{q}(s)$, the "Laissez-Faire" equilibrium prevails.*
- *if $q \leq \bar{q}(s)$, there exists no pure strategy equilibrium and in any mixed strategy equilibrium the entrant obtains a payoff equal to $\frac{1}{4}s(1-q)^2$.*

The formal proof is this proposition is relegated to the Appendix. A sketch of this proof goes as follows. Firstly, we derive firms' best response. Secondly, we identify the domain of parameters in which the "Laissez-Faire" equilibrium analysis still applies. Thirdly, we characterize a mixed strategy equilibrium where only the entrant firm mixes over two atoms and characterize the associated payoffs. Finally, we show that when the quota is tight, the former mixed strategies equilibrium fails to exist but we are able to prove that in any equilibrium (involving non-degenerated mixed strategies for the two firms) the entrant obtains the payoff $\frac{1}{4}s(1-q)^2$, which is its minmax payoff. The equilibrium payoffs characterized in Proposition 2 is clearly reminiscent of earlier results established in the case of homogeneous products. In particular Kreps and Scheinkman (1983) exhibit mixed strategy equilibria where the large capacity firm always earns its minmax payoff.

3.3 Homogeneous Products

We analyze now the equilibrium of the pricing subgame where firms sell identical qualities, i.e. $s = 1$. In this case, the vertical differentiation model degenerates into a standard Bertrand-Edgeworth duopoly with a market demand equal to $D(p) = 1 - p$, but with a quantity constraint q for one firm. Levitan and Shubik (1972) study such a game under efficient rationing. Defining $\lambda(q) \equiv \frac{1-\sqrt{q(2-q)}}{2}$, they show:

Lemma 2 *In a Nash equilibrium of the pricing game where $s_i = s = 1$, firms play a mixed strategy with common support $\left[\lambda(q), \frac{1-q}{2}\right]$ and cumulative distributions $F_e(p) = 1 - \frac{\lambda(q)}{p}$ and $F_i(p) = \frac{p(1-p)-\lambda(q)(1-\lambda(q))}{pq}$.*

Observe that $F_i(\lambda(q)) = 0$, $F_i\left(\frac{1-q}{2}\right) = 1$, $F_e(\lambda(q)) = 0$ and $F_e\left(\frac{1-q}{2}\right) < 1$, thus only the entrant has an atom at the upper price $\frac{1-q}{2}$. In this equilibrium, the incumbent's profit is $\Pi_i(q) = q\lambda(q)$ since at his lowest price he gets the whole demand $1-\lambda(q)$ thus sells q because $\lambda(q) < \frac{1-q}{2} < 1-q$ implies that his capacity constraint is binding. The entrant earns $\frac{1}{4}(1-q)^2$ because at his highest price, he receives the residual demand $1-q$. Notice last that this latter payoff is $\Pi_e(q,1)$, the limit of the equilibrium payoff obtained in Proposition 2 when product differentiation tends to zero.

3.4 Optimal Quality Choice for the Entrant

With the help of Proposition 2 and Lemma 2, we may now turn to the selection of an optimal quality by the entrant given the capacity restriction q. The analysis is illustrated on Figure 2.

Proposition 3 *In a SPE of* Γ_q*, the entrant selects* $s = 1$ *whenever* $q \leq q^* \equiv 1 - \frac{1}{2\sqrt{3}}$ *and* $s = \frac{4}{7}$ *otherwise.*

Proof Over the domain, $q > \bar{q}(s)$, where the "Laissez-Faire" equilibrium exists (see Figure 1), the best response in quality is given by the "Laissez-Faire" candidate $s = \frac{4}{7}$ (or $s_e = \overline{q}^{-1}(q)$ whenever $\frac{4}{7}$ lies outside the relevant domain). Whenever, $q \leq \bar{q}(s)$, the price equilibrium is in mixed strategies and the entrant's payoff is $\Pi_e(q,s) = \frac{s(1-q)^2}{4}$, so that the best response is obviously the top quality $s = 1$; we refer to this as the "imitation" strategy. To characterize the SPE of Γ_q, we compare the previous profits. Solving for $\frac{1}{48} = \Pi_e^*(\frac{4}{7}) = \Pi_e(q,1) = \frac{(1-q)^2}{4}$, we obtain the cut-off value $q^* \equiv 1 - \frac{1}{2\sqrt{3}} \simeq 71\%$. ■

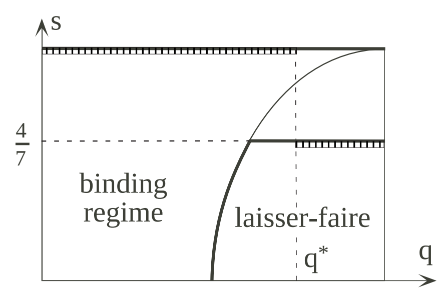

Figure 2: The quota-quality space

3.5 Optimal Capacity Restriction for the Regulator

We identify now the welfare maximizing capacity restriction. Notice first that if a regulator's objective was simply to ensure the provision of the best available quality by both firms, it would be sufficient to impose a capacity restriction at level $q^* \simeq 71\%$ (or any lower level). Although binding in the ensuing price game, this restriction is not unreasonable. In particular, it is larger than the equilibrium sales' level of the incumbent in the "Laissez-Faire" case ($\simeq 58\%$).

Similar to the case of the MQS, a capacity restriction might trigger different entry and quality choices from the challenger; two distinct regimes must be analyzed and compared. The intuition underlying the welfare comparison is nevertheless easy to grab. Over the domain where the capacity restriction induces quality imitation, a looser quota reduces industry profits as we approach the standard Bertrand equilibrium with zero profits. In other words, a looser quota generates a fiercer competition at the price stage and a greater consumer surplus. Computations show that the ensuing increase in consumer surplus dominates the loss of industry profits. Accordingly, the optimal capacity restriction is the loosest quota compatible with quality imitation.

Proposition 4 *The optimal capacity restriction for the regulator is* $q^* = 1 - \frac{1}{2\sqrt{3}}$.

Proof: For $q \leq q^*$, we know from Proposition 3 that the entrant chooses the highest quality and competition takes places in a market for a homogeneous good. In this equilibrium the incumbent profit is $\Pi_i(q) = q\lambda(q)$ while the entrant obtains $\Pi_e(q) = \frac{(1-q)^2}{4}$. We show in Lemma 3 of the appendix that market welfare, net of fixed cost, is $\overline{W}(q) = \frac{3}{8} + q\frac{4-q-2\sqrt{q(2-q)}}{8}$. This function is increasing and concave in q. Since the "Laissez-Faire" welfare is $\frac{3}{8}$, a capacity restriction $q \leq q^*$ yields a greater welfare and the optimal choice is thus q^*, the highest quota compatible with $s = 1$ in a SPE. Notice that welfare is $\overline{W}(q^*) \simeq 0.497$. ■

3.6 Comparing Capacity Restriction and MQS

We can now assess the respective merits of Capacity Restrictions and Minimum Quality Standards in our model of entry with sunk cost. Notice from Propositions 3 and 4 that the entrant's operating profits are exactly equal to $\frac{1}{48}$ at the optimal quota q^* as in the Laissez-Faire case. Therefore, the presence of the entry cost F does not constrain the government's possibilities, as compared to the case of a MQS policy. However, the optimal capacity restriction does not yield the first best welfare of $\frac{1}{2}$ whereas the MQS does at the limit where sunk cost is nil. Formally, we may state:

Proposition 5 *There exists a threshold fixed cost $\underline{F}$ such that for $F > \underline{F}$, a capacity restriction induces a higher market welfare than a minimum quality standard.*

Proof From Proposition 1, we know that the maximum welfare with a MQS is $\underline{W}(z^*(F)) - F$ where $\underline{W}(z^*(F))$ is a decreasing function of F. From Proposition 4, we know that the maximum welfare with a CR is $\overline{W}(q^*) - F$. The cut-off is thus the solution $\underline{F} \simeq 4.76 \times 10^{-3}$ of $\underline{W}(z^*(F)) = \overline{W}(q^*)$. ■

The economic intuition underlying our result is straightforward. A capacity restriction relaxes price competition by inducing a less aggressive behavior of the constrained firm, here the incumbent. Recall then that in a vertically differentiated duopoly, one firm selects a low quality in order to relax competition. However, in the presence of the capacity restriction this is no longer necessary because the capacity restriction is a more powerful instrument to reduce competition. The incumbent is less tempted to undercut since at some point this does not increase sales. On the other hand, the entrant benefits from the possibility of retreating on the residual market where it enjoys quasi-monopoly profits.

The entrant thus looses any incentive to downgrade quality and both firms end up selecting a high quality. Moreover, because price competition is softer, equilibrium profits for any quality pair tend to be larger. There exists however a limit to the effective level of the capacity restriction. If it is set at a too loose level, the entrant enjoys an extremely limited protected market and therefore prefers to differentiate optimally. The mechanism at work may therefore be summarized as follows: the quota alters the payoffs in the second stage in such a way that the entrant's incentive at the first stage are put in the "right" direction, i.e. towards quality upgrades.

The MQS mechanism on the other hand, directly constrains the firms' strategy space at the quality stage. By definition, in order to be effective, the MQS must run *against* firms' incentives. By leaving less room for differentiation, the MQS undermines firms' profits in equilibrium and therefore impedes entry. As shown in Proposition 5, it is only when the entry costs are negligible that the government prefers the MQS to a capacity restriction. In this case indeed, the fact that operating profits sharply decrease because of a very high MQS is not a concern anymore. By contrast, the residual market power that must be left to firms in order to induce quality upgrades does not depend on F.

4 Conclusion

In vertically differentiated industries, MQS are often used to control for quality provision. The literature has however revealed that in highly concentrated industries, MQS may have adverse effects from a welfare point of view, because they undermine firms' profitability. In the present paper, we have investigated an alternative route. Within a very simple model, we have shown that a capacity restriction might be more efficient than MQS.

Our formal model is quite specific, although it should be stressed that it is quite in line with the models developed in the received literature on MQS. Several generalizations can be contemplated. Firstly, the introduction of positive quality costs that would be sunk before price competition does not alter our qualitative conclusions. The presence of the capacity restriction at the price competition stage relaxes competition in exactly the same way, so that there are no strategic motives left for low quality selection. However, we do not expect minimal differentiation anymore. Indeed, the entrant remains likely to opt for a lower quality in order to save on sunk quality costs, exactly as he would do under Cournot competition. Still, the average quality bought by consumers increases and industry welfare increases as well.[12] Second, one may question the robustness of our results to the exogenously imposed quality hierarchy. Relying on Boccard and Wauthy (1998), we may actually claim that the present results remain valid if we do not impose any exogenous quality hierarchy between the entrant and the incumbent.[13] A third avenue regards the mode of competition. As we show in Lemma 4 in the appendix, comparable conclusions are reached under Cournot competition as well.

All in all, the driver of our result is robust and derives simply from the intrinsic ability of quantitative restraints to relax price competition. In vertically differentiated industries, this almost immediately implies that firms do not need to relax competition by differentiating products. Accordingly, average quality may rise. Regarding quality selection, the chief merit of the capacity restriction is thus quite clear: it gives to all firms an incentive to select a high quality for their products and therefore destroys the incentives towards strategic quality under-provision. A natural extension that comes to mind consists in considering a larger stage-game in which firms who endogenously select quality and capacity levels. However, this is an extremely challenging task, because the general structure of mixed strategy equilibria when two firms are capacity constrained is yet to be studied. The results we report here make a first step in the relevant direction but they do not immediately generalize to all price subgames with two active capacity constraints.

Appendix

Price Equilibrium under Laissez-Faire

[12] See Boccard and Wauthy (1998) for a more detailed analysis.

[13] In a set-up where one firm may simultaneously decide on quality and capacity levels, Boccard and Wauthy (2009) also establish results that point to the robustness of the quality imitation effect resulting from Bertrand-Edgeworth competition.

Firms' profits at the last stage of the game are

$$\Pi_e(p_i, p_e) = p_e D_e(p_i, p_e) \quad \text{and} \quad \Pi_i(p_i, p_e) = p_i D_i(p_i, p_e) \tag{9}$$

Consequently, we limit ourselves to an informal (and mainly graphical) argument. The payoffs are continuous and give rise to continuous best response functions, as illustrated on Figure 3. Notice in particular that the striped area characterizes the prices constellation for which both firms enjoy a positive demand. The frontiers of this region are therefore given by the different critical values defining the kinks in equations (2) and (3).

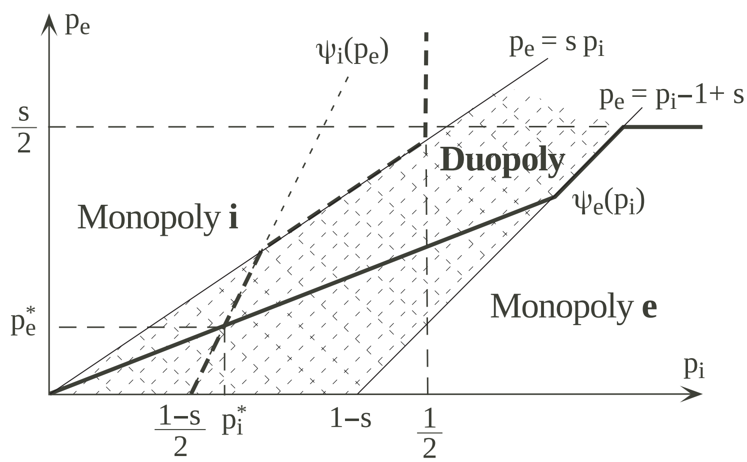

Figure 3: The price space

Computing the first order conditions on profit functions defined in equation (9), $\frac{\partial \pi_k}{\partial p_k} = 0$, $k = i, e$ and again taking non-negativity constraints into account, the firms' best responses can be expressed as follows:

$$\psi_e(p_i) = \min\left\{\max\left\{\frac{p_i s}{2}, p_i + 1 - s\right\}, \frac{s}{2}\right\} \tag{10}$$

and

$$\psi_i(p_e) = \min\left\{\max\left\{\frac{1 - s + p_e}{2}, \frac{p_e}{s}\right\}, \frac{1}{2}\right\} \tag{11}$$

Notice that in each best response, the first term is relevant for an interior solution, then the second corresponds to a kink which becomes relevant when we hit the non-negativity constraint. Last, the third term identifies the monopoly price which is a relevant candidate whenever the other's price is so high that its demand is equal to zero.

The unique price equilibrium given by the intersection of the best responses curves (10–11) is

$$p_e^* = \frac{s(1-s)}{4-s} \quad \text{and} \quad p_i^* = \frac{2(1-s)}{4-s} \tag{12}$$

Proof of Lemma 1 *If quality is not costly and the incumbent sells quality $s_i = 1$, then whenever $F \leq \frac{1}{48}$, the entrant enters and optimally differentiates by selecting quality $\frac{4}{7}$. The price equilibrium of the continuation game is unique and in pure strategies.*

Recall that

$$D_e(p_e, p_i) = \begin{cases} 1 - \frac{p_e}{s} & if \quad p_e \leq p_i - 1 + s \\ \frac{p_i s - p_e}{s(1-s)} & if \quad p_i - 1 + s \leq p_e \leq p_i s \\ 0 & if \quad p_e \geq p_i s \end{cases} \tag{13}$$

$$D_i(p_e, p_i) = \begin{cases} 1 - p_i & if \quad p_i \leq \frac{p_e}{s} \\ 1 - \frac{p_i - p_e}{1-s} & if \quad \frac{p_e}{s} \leq p_i \leq p_e + 1 - s \\ 0 & if \quad p_i \geq p_e + 1 - s \end{cases} \tag{14}$$

and that profits are $\Pi_e(p_i, p_e) = p_e D_e(p_i, p_e)$ and $\Pi_i(p_i, p_e) = p_i D_i(p_i, p_e)$.

The solution to $\frac{\partial \Pi_e}{\partial p_e} = 0$ over the range where both demands are non-negative is $\psi_e(p_i) \equiv \frac{p_i s}{2} \leq p_i s$; thus, the low quality best response function is $\phi_e(p_i) = \psi_e(p_i)$. In the incumbent monopoly region ($p_e > p_i s$), the incumbent's best response is the monopoly price $\frac{1}{2}$ which is feasible if and only if $p_e > \frac{s}{2}$. Otherwise, Π_i is strictly increasing in the monopoly region and we always reach the duopoly region where the profit is $p_i \left[1 - \frac{p_i - p_e}{1-s}\right]$ leading to a candidate best response $\psi_i(p_e) \equiv \frac{p_e + 1 - s}{2}$. Whenever $p_e \leq \frac{s(1-s)}{2-s}$ then $\psi_i(p_e) \leq \frac{p_e}{s}$ meaning that ψ_i

is the best response, otherwise it is the frontier price $\frac{p_e}{s}$ which is optimal. As we have $\frac{s(1-s)}{2-s} < \frac{s}{2}$, the (kinked) best response of firm h is

$$\phi_i(p_e) = \begin{cases} \psi_i(p_e) & if \quad p_e \leq \frac{s(1-s)}{2-s} \\ \frac{p_e}{s} & if \quad \frac{s(1-s)}{2-s} \leq p_e \leq \frac{s}{2} \\ \frac{1}{2} & if \quad \frac{s}{2} \leq p_e \end{cases} \tag{15}$$

As one can see on Figure 3 in the text p.21, the Laissez-Faire equilibrium $(p_e^*, p_i^*) = \left(\frac{s(1-s)}{4-s}, \frac{2(1-s)}{4-s}\right)$ is given by the intersection of ψ_e and ψ_i.

In the quality stage we have $\Pi_i(s) \equiv p_i^* D_i^* = \frac{4(1-s)}{(4-s)^2}$ and $\Pi_e(s) \equiv p_e^* D_e^* = \frac{s(1-s)}{(4-s)^2}$. It is a matter of calculations to check that Π_e reaches its maximum for $s = \frac{4}{7}$. ■

Proof of Proposition 2

Step 1: It is clear from eq. (7) that the best response of the incumbent over the binding regime is the largest available price $\beta^{-1}(p_e)$. Using the continuity of payoffs, which results from the continuity of sales function, we note that this optimal price is weakly dominated by the best response of the competitive (non binding) regime. The candidate best reply in that regime has been previously characterized as $\psi_i(p_e) = \frac{p_e+1-s}{2}$, so that whenever this later price belongs to the competitive regime, it is the best reply of the incumbent. Formally, we obtain the best response

$$\phi_i(p_e) = \begin{cases} \psi_i(p_e) & \text{if} \quad p_e \leq \bar{p}_e \\ \beta^{-1}(p_e) & \text{if} \quad p_e \geq \bar{p}_e \end{cases} \tag{16}$$

where $\bar{p}_e \equiv \max\{0, (2q-1)(1-s)\}$ solves $\psi_i(p_e) = \beta^{-1}(p_e)$. The best response of the incumbent is displayed on Figure 4 in dotted bold face; it is continuous with a kink at $\bar{p}_e$.[14] The non negativity constraint (NNC) $S_i = D_i = 0$ displayed on Figure 4 is defined by equation $p_i = p_e + 1 - s$.

In the binding regime, the entrant benefits from a monopoly position over a protected market of size $1-q$, his profit is $\pi_e = (1 - q - \frac{p_e}{s_e})p_e$ and reaches a

[14]The β^{-1} line crosses the frontier between duopoly and monopoly for the incumbent at $p_i = 1-q$ and $p_e = s(1-q) = 2p_e^s$; for larger p_e it becomes a vertical.

maximum of $\Pi_e(q,s) \equiv \frac{s(1-q)^2}{4}$ at price $p_e^s \equiv \frac{(1-q)s}{2}$, which will later be referred to as the security price. Notice that $\Pi_e(q,s)$ defines the minmax payoff of firm e in the corresponding price subgame. In the competitive regime, the best response candidate is the unregulated candidate $\psi_e(p_i) = \frac{p_i s}{2}$. The associated payoff is $\Pi_e(\psi_e(p_i), p_i) = \frac{sp_i^2}{4(1-s)}$ which is increasing in p_i. It then remains to solve

$$\Pi_e(q,s) = \frac{sp_i^2}{4(1-s)} \Leftrightarrow p_i = \mu(q,s) \equiv (1-q)\sqrt{1-s}$$

in order to obtain the entrant's best response correspondence:[15]

$$\phi_e(p_i) = \begin{cases} p_e^s & \text{if} \quad p_i \leq \mu(q,s) \\ \psi_e(p_i) & \text{if} \quad p_i \geq \mu(q,s) \end{cases} \tag{17}$$

Step 2: Since $\phi_e(.)$ is discontinuous at $\mu(q,s)$, the existence of a pure strategy equilibrium is not ensured. There are however four pure strategy candidates corresponding to the combinations "binding" and "competitive" among the two firms.

Firstly, we have the "Laissez-Faire" equilibrium (p_i^*, p_e^*) candidate where both firms are in the competitive regime; it is indeed an equilibrium if $p_i^* > \mu(q,s) \Leftrightarrow q > \bar{q}(s) \equiv 1 - \frac{2\sqrt{1-s}}{4-s}$.

The second candidate is when the entrant is in the competitive regime while the incumbent sells at the quota level; it is the intersection of $\psi_e(p_i)$ and $\beta^{-1}(p_e)$ at $\hat{p}_i = 2(1-q)\frac{1-s}{2-s}$. However this is not a valid candidate because one can check that $\hat{p}_i < \mu(q,s)$ holds true, thus the relevant branch of the best reply $\phi_e(p_i)$ is actually p_e^s. The other two candidates for a pure strategy equilibrium are when the horizontal line at p_e^s crosses either $\beta^{-1}(.)$ or $\phi_i(.)$; both can be dismissed because the jump at $\mu(q,s)$ will always occur inside the binding area i.e., before the relevant intersection.

Step 3: Suppose $q < \bar{q}(s)$. As illustrated on Figure 4, there exists no pure strategies equilibrium. A natural candidate is proposed by Krishna (1989): the incumbent

[15] ϕ_e is single valued except at $\mu(q,s)$ where it admits two values.

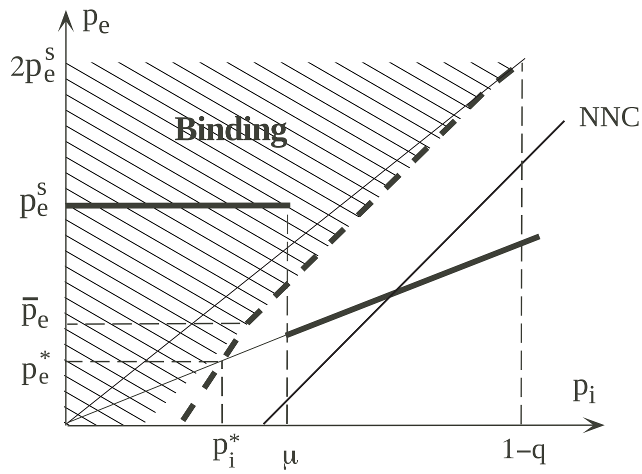

Figure 4: The price space with a mild capacity restriction

plays the pure strategy $\mu(q,s)$ and the entrant randomizes over the pair of prices p_e^s and $\psi_e(\mu(q,s))$. By definition of $\mu(q,s)$, the entrant is indifferent between p_e^s and $\psi_e(\mu(q,s))$; any mixture over these two prices yields the same payoff. We may then formally compute the weights that make $\mu(q,s)$ a best response for the incumbent.

Let α be the weight on p_e^s. When facing p_e^s, the sales of firm i are $S_i = q$ while they are $S_i = 1 - \frac{p_i - \psi_e(\mu)}{1-s}$ when facing $\psi_e(\mu)$. The expected profit is thus

$$\pi_i = p_i \left[\alpha q + (1-\alpha)\left(1 - \frac{p_i - \psi_e(\mu)}{1-s}\right)\right] \tag{18}$$

and is maximum when $\alpha q + (1-\alpha)\left(1 - \frac{2p_i - \psi_e(\mu)}{1-s}\right) = 0$ i.e., for

$$p_i = \frac{\psi_e(\mu)}{2} + \frac{1-s}{2}\left(\frac{\alpha q}{1-\alpha} + 1\right) = \frac{\mu s}{4} + \frac{1-s}{2}\left(\frac{\alpha q}{1-\alpha} + 1\right) \tag{19}$$

Now, in equilibrium, α is such that this best reply is exactly μ hence

$$\alpha = \frac{\frac{\mu}{2}\left(\frac{4-s}{1-s}\right) - 1}{\frac{\mu}{2}\left(\frac{4-s}{1-s}\right) - 1 + q} < 1.$$

Observe that $\alpha > 0 \Leftrightarrow (4-s)(1-q)\sqrt{1-s} > 2(1-s) \Leftrightarrow q < \bar{q}(s)$ which is true in the present case. A necessary condition for this equilibrium to exist is that

$D_i(\mu, \psi_e(\mu)) > 0$, i.e. the incumbent receives a positive demand, for otherwise he would reduce his price to get some demand and some profit. Solving this inequality for q, we obtain the restriction $q > \underline{q}(s) \equiv 1 - \frac{2\sqrt{1-s}}{2-s}$.

Recall then that the entrant's equilibrium profit can be computed at any of the prices in the support of his equilibrium strategy, and for instance at the security price p_e^s where his payoff is already known to be $\Pi_e(q,s) \equiv \frac{s(1-q)^2}{4}$.

Step 4 Whenever $q \leq \underline{q}(s)$, the semi-mixed strategy equilibrium identified in step 3 does not exist. However, a fully mixed strategy equilibrium must exist because firms' payoffs are continuous in prices. We show that in every possible mixed strategy equilibrium, the entrant earns $\Pi_e(q,s)$. Figure 5 depicts a configuration where the non-negativity constraint (NNC) is binding for the mixed strategy equilibrium candidate identified in Step 3. Recall that the frontier between the binding and non-binding quota regimes is identified with $\beta(.)$. Best responses are drawn in bold face. For $j = i,e$, we denote by F_j the firm j's mixed strategy in a Nash equilibrium.

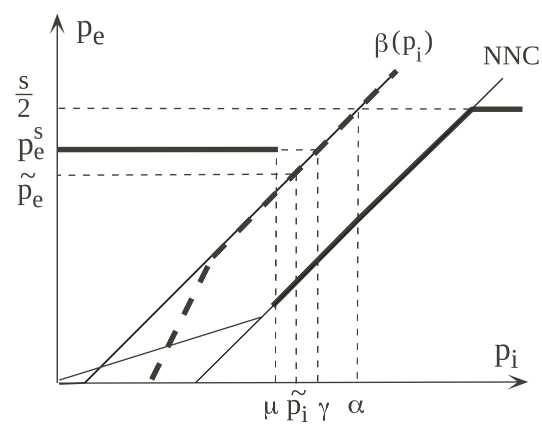

Figure 5: Best responses in prices under a severe capacity restriction

We first show that players supports are quite limited. Observe that, by construction of the best response, the entrant's profit is decreasing in own price over $\left[\frac{s}{2}, 1\right]$ whatever p_i may be. Hence, $\Pi_e(p_e, F_i) = \int \Pi_e(p_e, p_i) dF_i(p_i)$, the average over F_i is likewise decreasing over the same range so that the support of F_e has to be included in $\left[0, \frac{s}{2}\right]$. For $p_e \in \left[0, \frac{s}{2}\right]$, the incumbent's profit is decreasing in own price over $[\alpha, 1]$, hence the average over F_e is likewise decreasing over the same range so that the support of F_i is included in $[0, \alpha]$. For $p_i \in [0, \alpha]$, the entrant's profit is decreasing in own price over $\left[p_e^s, \frac{s}{2}\right]$ (because he needs not consider the area on the right of the NNC), hence the average over F_i is likewise decreasing over the same range so that the support of F_e is included in $[0, p_e^s]$. By the same token the support of F_i is included in $[0; \gamma]$.

Let $\tilde{p}_e$ be the supremum of the support of F_e and $\tilde{p}_i = \beta^{-1}(\tilde{p}_e)$. We claim that $\tilde{p}_e = p_e^s$. If not, the previous reasoning applies again telling us that Π_i is decreasing over $[\tilde{p}_i, \gamma]$ for every $p_e \in [0, \tilde{p}_e]$, hence the incumbent does not play prices above $\tilde{p}_i$ in equilibrium. Now recall that in a mixed strategy equilibrium the payoff of a player can be computed at any of the prices belonging to the support of his optimal strategy; let us then consider $\tilde{p}_e$ for the entrant. For any $p_i \in [0, \tilde{p}_i]$, the incumbent is constrained by the quota so that the entrant is a monopoly over a market of size $1 - q$, hence her optimal behavior is to try to reach the price p_e^s. This stands in contradiction to the fact that $\tilde{p}_e$ is the highest optimal price. We have thus proven that $\tilde{p}_e = p_e^s$ and as a consequence that the equilibrium payoff is $\Pi_e(\tilde{p}_e, F_i) = \frac{s}{4}(1-q)^2$ since the support of F_i is included in $[0, \gamma]$. ■

The Social Welfare Funtion in Game Γ_q

Lemma 3 *In Γ_q, market welfare, net of fixed cost, is* $\overline{W}(q) = \frac{3}{8} + q\frac{4-q-2\sqrt{q(2-q)}}{8}$.

Proof The surplus of the consumer with type $x \in [0, 1]$ is best understood by separating 2 cases:

- if $x > 1 - q$, then $x > p_e$ because $p_e \leq \frac{1-q}{2}$. The incumbent price p_i is the lowest with probability $F_i(p_e)$ in which case the consumer buys at the price p_i (because $x > p_e > p_i$ and the incumbent is not constrained) so that we need to compute an expectation. With complementary probability, the consumer buys at the entrant, thus the surplus of consumer x is

$$H(x,p_e) \equiv (x-p_e)(1-F_i(p_e)) + \int_{\lambda(q)}^{p_e} (x-p_i)dF_i(p_i)$$

- if $x < 1-q$, the consumer is rationed by the incumbent; then either $x < p_e$ so that he does not buy at all, or $x > p_e$ and he buys from the entrant deriving a surplus of $x - p_e$.

Integrating with respect to the distribution of the entrant's prices, we have three cases according to the respective positions of x and the upper price limit:

- if $x < \frac{1-q}{2}$, $W_a(q,x) \equiv \int_{\lambda(q)}^{x} (x-p_e)dF_e(p_e)$
- if $\frac{1-q}{2} < x < 1-q$, $W_b(q,x) \equiv \int_{\lambda(q)}^{\frac{1-q}{2}} (x-p_e)dF_e(p_e) + \left(x - \frac{1-q}{2}\right)\left(1 - F_e\left(\frac{1-q}{2}\right)\right)$
- if $1-q < x$, $W_c(q,x) \equiv \int_{\lambda(q)}^{\frac{1-q}{2}} H(x,p_e)dF_e(p_e) + H\left(x, \frac{1-q}{2}\right)\left(1 - F_e\left(\frac{1-q}{2}\right)\right)$

Integrating with respect to the uniform distribution of consumers over the range of potential buyers i.e., $x \geq \lambda(q)$, we obtain the consumer surplus expression:

$$\begin{aligned} W_C(q) &\equiv \int_{\lambda(q)}^{\frac{1-q}{2}} W_a(q,x)dx + \int_{\frac{1-q}{2}}^{1-q} W_b(q,x)dx + \int_{1-q}^{1} W_c(q,x)dx \\ &= \frac{1}{8} + q\frac{4-3q+2\sqrt{q(2-q)}}{8} \end{aligned}$$

simplifies is an increasing and concave function. Observe that $W_C(1) = \frac{1}{2}$, is the market welfare at the outcome of Bertrand competition between two identical products where no consumer refrains from buying, all consumers buy the best available quality and firms capture no rent. The market welfare summing consumer surplus and producers surplus is

$$\overline{W}(q) = W_C(q) + \Pi_i(q) + \Pi_e(q) = \frac{3}{8} + q\frac{4-q-2\sqrt{q(2-q)}}{8} > \frac{3+q}{8} \quad \blacksquare$$

Lemma 4 *We address here the case where firms compete in quantity in the last stage of game. We show that quality imitation occurs in equilibrium.*

When the demands (2) and (3) are positive, we have

$$q_i = 1 - \frac{p_i - p_e}{1-s} \quad \text{and} \quad q_e = \frac{p_i s - p_e}{s(1-s)} \tag{20}$$

so that the inverse demands characterizing Cournot competition are given by

$$p_i = 1 - q_i - q_e s \quad \text{and} \quad p_e = (1 - q_i - q_e)s \tag{21}$$

The best replies in quantities are immediately derived as

$$BR_i^c(q_e) \equiv \frac{1-q_e s}{2} \quad \text{and} \quad BR_e^c(q_i) \equiv \frac{1-q_i}{2} \tag{22}$$

The unconstrained Cournot equilibrium is thus

$$q_e^c(s) \equiv \frac{1}{4-s} \quad \text{and} \quad q_i^c(s) \equiv \frac{2-s}{4-s}. \tag{23}$$

leading to equilibrium prices

$$p_e^c = \frac{s}{4-s} \quad \text{and} \quad p_i^c = \frac{2-s}{4-s} \tag{24}$$

Notice that q_e^c is increasing with s while q_i^c is decreasing. The entrant's profits at the Cournot equilibrium are $\pi_e^c(s) \equiv \frac{s}{(4-s)^2}$ and since $\frac{\partial \pi_e^c}{\partial s} = \frac{4+s}{(4-s)^3} > 0$, the optimal choice for the low quality firm is imitation. $\blacksquare$

References

Albano G. L. and Lizzeri A. Strategic certification and provision of quality. *International Economic Review*, 42(1):267–283, 2001.

Auriol E. Deregulation and quality. *International Journal of Industrial Organization*, 16(2):169–194, 1998.

Boccard N. and Wauthy X. Quality choice, sales restriction and the mode of competition. *The Manchester School*, 74(1):64–84, 2006.

Boccard N. and Wauthy X. Import restraints foster minimum differentiation in vertically differentiated industries. Technical Report 9818, CORE, 1998.

Boccard N. and Wauthy X. Entry accommodation under multiple commitment strategies: judo economics revisited. Technical Report 50, CORE, 2009.

Cabral C. C., Kujal P., and Petrakis E. Incentives for cost reducing innovations under quantitative import restraints. *Annales d'économie et de statistique*, (49-50):479–493, 1998.

Chipty T. and Witte A. D. An empirical investigation of firms responses to minimum standards regulations. Technical Report 6104, NBER, 1997.

Choi C. J. and Shin H. S. A comment on a model of vertical product differentiation. *Journal of Industrial Economics*, 40(2):229–231, 1992.

Crampes C. and Hollander A. Duopoly and quality standards. *European Economic Review*, 39(1):71–82, 1995.

Davidson C. and Deneckere R. Long-run competition in capacity, short-run competition in price and the cournot model. *RAND Journal of Economics*, 17(3): 404–415, 1986.

Furth D. and Kovenock D. Price leadership in a duopoly with capacity constraints and product differentiation. *Journal of Economics*, 57(1):1–35, 1993.

Glass A. J. Price discrimination and quality improvement. *Canadian Journal of Economics*, 34(2):549–569, 2001.

Kreps D. and Scheinkman J. Quantity precommitment and bertrand competition yields cournot outcomes. *Bell Journal of Economics*, (14):326–337, 1983.

Krishna K. Trade restrictions as facilitating practices. *Journal of International Economics*, 26(3-4):251–270, 1989.

Levitan R. and Shubik M. Price duopoly and capacity constraints. *International Economic Review*, 13:111–122, 1972.

Lutz S., Lyon T. P., and Maxwell J. W. Quality leadership when regulatory standards are forthcoming. *Journal of Industrial Economics*, 48(3):331–348, 2000.

Maxwell J. W. Minimum quality standards as a barrier to innovation. *Economics Letters*, 58(3):355–360, 1998.

Mussa M. and Rosen S. Monopoly and product quality. *Journal of Economic Theory*, (18):301–317, 1978.

Rhoades D. L. and Waguespack B. Judging a book by it's cover: the relationship between service and safety quality in us national and regional airlines. *Journal of Air Transport Management*, 6(2):87–94, 2000.

Ronnen U. Minimum quality standards, fixed costs, and competition. *RAND Journal of Economics*, 22(4):490–504, 1991.

Scarpa C. Minimum quality standards with more than two firms. *International Journal of Industrial Organization*, 16(5):665–676, 1998.

Tirole J. The theory of industrial organization. MIT press, 1988.

Valletti T. M. Minimum quality standards under cournot competition. *Journal of Regulatory Economics*, 18(3):235–245, 2000.