Risk-sharing networks and farsighted stability [†]

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Abstract

Evidence suggests that in developing countries, agents rely on mutual insurance agreements to deal with income or expenditure shocks. This paper analyzes which risk-sharing networks can be sustained in the long run when individuals are farsighted, in the sense that they are able to forecast how other agents would react to their choice of insurance partners. In particular, we study whether the farsightedness of the agents leads to a reduction of the tension between stability and efficiency that arises when individuals are myopic. We find that for extreme values of the cost of establishing a mutual insurance agreement, myopic and farsighted agents form the same risk-sharing networks. For intermediate costs, farsighted agents form efficient networks while myopic agents don't.

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1 Introduction

In this paper, we study the formation of risk-sharing networks. There are regions in developing countries where the access to a formal insurance market is limited. Some villages lack for instance institutions that can enforce contracts or repayments of loans. Economic fluctuations, due to climate shocks, crop pests, illness or funeral expenditures are important in those low income areas. Informal risk-sharing appears to be one of the prominent strategy used to cope with these shocks (see the survey of Alderman and Paxson, 1994). That is, households in need receive help from others, in the form of free loans or transfers. A growing empirical literature (see Fafchamps, 1992; Grimmard, 1997; Fafchamps and Lund, 2003; De Weerdt and Dercon, 2006) has shown that a fully efficient risk-pooling equilibrium is not reached: risk-sharing does not take place within exogenous group such as the village, but rather within networks involving agents having common characteristics (neighborhood, professional or religious affiliation, kinship, etc).

Most of the theoretical papers on informal risk-sharing in developing countries assume that no binding agreement can be enforced. In this context, if the risk occurs only once, the fortunate agent has no incentive to transfer money ex-post. However, this effect disappears in a dynamic setting where multiple shocks are expected to occur since agents who transfer money today may expect to be reciprocated at a future date. This literature offers a theoretical argument to explain the observed lack of complete income pooling at the village level by analyzing the transfer schemes which are such that each agent is willing to conform to the agreement once the uncertain income shock occurs.[1]

[1]Kimball (1988) has shown that if individuals are sufficiently patient, then some first-best allocation, i.e. allocations that would be efficient if agents had the possibility to commit to a transfer scheme, can be implemented as a subgame perfect equilibrium. Coate and Revallion (1993) have studied the symmetric two-player model by restricting their analysis to stationary transfers. They have improved upon Kimball (1988) in that they have endogenized the amount of transfers while previous work had considered only the extreme cases of complete income pooling or no transfer at all. Kocherlakota (1996) has further analysed the case of impatient agents and has shown that the Pareto-undominated subgame perfect allocations imply a positive correlation between individual consumption and current and lagged income. Ligon, Thomas and Worrall (2002) have shown that allowing transfers to depend upon history of transfers is payoff improving for the agents. Genicot and Ray (2003) have further studied this problem by adding the possibility that groups of agents jointly deviate from the prescribed transfer scheme. Finally, Bloch, Genicot and Ray (2005) have adapted previous work for situations where transfers occur through a network

Platteau (2002) has argued that risk-sharing usually occurs among relatives and even if the institutional context does not provide the tools to enforce contracts, agents involved in a risk-sharing relationship may be committed to the agreement because the social norm imposes it. Bramoullé and Kranton (2007a) have developed a model, where agents establish their informal insurance relationships endogenously, assuming that linked pairs can commit to share equally their income. They have considered agents who are ex ante homogeneous, but differ through their position in the network.[2] They have shown that the efficient risk-sharing networks are such that each agent is indirectly connected to the others, involving the maximal level of insurance in the population, and that networks formed by myopic agents connect fewer individuals than the efficient ones. They have thus provided another theoretical explanation of the observation that informal insurance does not occur at the village level.

Empirical studies support the idea that mutual risk-sharing agreements are formed endogenously. For instance, Rosenweig and Stark (1989) have observed that marriages between households from different regions in India occur to diversify geographically the risks.[3] Dekker (2004) has studied the endogenous formation of risk-sharing networks in four resettlement villages in rural Zimbabwe. She has found that in a social environment where blood relatives are scarce, resettled households have strongly invested in activities to establish links with surrogate relatives. Comola (2008) has observed that the structure of the network, that is the social position of an agent with respect to the others, is critical to understand the choice of risk-sharing partners. Based on data on the village Nyakatoke in Tanzania, she has

rather than through a group.

[2] There have been other attempts to model the formation of informal network in the spirit of Jackson and Wolinsky (1996), where direct links involve benefits and costs, while indirect links affect positively or negatively the agents depending on the nature of the network externalities. Bramoullé and Kranton (2007b) have studied the formation of risk-sharing networks among agents living in two different villages, assuming that the shock to the income of households is village specific. Comola (2008) has proposed a model of network formation, where benefits and costs to links formation are heterogeneous. Krishnan and Sciubba (2008) have extended the co-author model of Jackson and Wolinsky (1996) to study bilateral labor exchange agreements among heterogeneous agents.

[3] Grimmard (1997) has found evidence of transfers and migrations in Côte d'Ivoire supporting this idea.

found that "not only the characteristics of direct friends, but also the characteristics of indirect contacts are taken into account when a link is created".

This paper analyzes which pattern of insurance relationships emerges in the long run when agents are farsighted, rather than myopic, in the sense that they are able to forecast how other agents would react to their choice of partners. In his survey of models of network formation, Jackson (2005) has mentioned that farsightedness is an important consideration in some appropriate context. He has stated that "in large networks it might be that players have very little ability to forecast how the network might change in reaction to the addition or deletion of a link. In such situations the myopic solutions are quite reasonable. However, if players have very good information about how others might react to changes in the network, then these are things that one wants to allow for either in the specification of the game or in the definition of the stability concept". To our knowledge, no existing work has attempted to establish whether agents are farsighted or not when creating their network in rural areas of developing countries. However, we believe that the key ingredients mentioned by Jackson (2005) for farsightedness to matter are present in this framework: our focus is on small communities, where agents have good information about each other. Agents in the model of Bramoullé and Kranton (2007a) are strategic: they establish links with other agents, anticipating that these connections might be profitable in the future if they face negative income shocks. In this paper, we assume that agents are a bit more strategic: in addition to forming connections in anticipation of likely future negative shocks, they also realize that their choice of partners may determine others' choices of partners. Such anticipation is consistent with Comola (2008)'s observation that the full architecture of bilateral agreements determines the incentives for a pair of agents to establish a partnership. We adopt the notion of pairwise farsightedly stable set due to Herings, Mauleon and Vannetelbosch (2009) to determine which networks are formed by farsighted agents.[4] We find that for small costs of establishing and maintaining a partnership, farsighted agents may form efficient networks that involve full income pooling while myopic agents form networks connecting fewer individuals. Two mechanisms explain this result: (i) Farsighted agents belonging to small groups may decide to create new partnerships

[4] Other approaches to farsightedness in network formation are suggested by the work of Xue (1998), Herings, Mauleon, and Vannetelbosch (2004), Mauleon and Vannetelbosch (2004), Page, Wooders and Kamat (2005), Dutta, Ghosal, and Ray (2005), and Page and Wooders (2009).

that are not directly profitable to them, because they realize that other partners will further join this bigger and more attractive group. In other words, the farsightedness of the agents may solve a coordination problem. (ii) Farsighted agents may refrain from deleting costly links if they belong to a big group, as they understand that this may induce others to rearrange their partnerships in a way that deters the myopic incentives to delete the link at first. We have already mentioned that empirical studies have revealed that risk-sharing occurs among agents having common characteristics. Farsightedness may be a factor rationalizing this observation.

The paper is organized as follows. In Section 2 we introduce some notations and definitions for networks, and we present the model of risk-sharing networks of Bramoullé and Kranton (2007a). In Section 3, we investigate the formation of risk-sharing networks when agents are myopic. Section 4 provides a characterization of the pairwise farsightedly stable set of risk-sharing networks. In Section 5, we analyze more in detail the formation of risk-sharing networks when agents have a quadratic utility function. In Section 6, we conclude.

2 Model and notation

Networks

A network (N, g) is defined by a set of agents $N = \{1, \ldots, n\}$ and a list g of which pairs of individuals among the agent set N are linked to each other. For sake of notation we simply use the set of links g to refer to the network when the player set N is fixed. The network relationships are reciprocal and the network is thus modelled as a non-directed graph. Individuals are the nodes in the network and links indicate bilateral relationships between individuals. We write $ij \in g$ to indicate that i and j are linked under the network g. Let g^N be the collection of all subsets of N with cardinality 2, so g^N is the complete network. The set of all possible networks on N is denoted by $\mathbb{G}$ and consists of all subsets of g^N. The network obtained by adding the link ij to an existing network g is denoted $g + ij$ and the network that results from deleting the link ij from an existing network g is denoted $g - ij$. For any network g, let $N(g) = \{i \in N \mid \exists\, j \text{ such that } ij \in g\}$ be the set of agents who have at least one link in the network g. The degree of agent i in a network g is the number of links that involve that agent: $d_i(g) = \#\{j \in N \mid ij \in g\}$. The total number of links of a network g is given by $d(g) = \Sigma_{i \in N} d_i(g)/2$. A path in a network $g \in \mathbb{G}$ between i and

j is a sequence of agents $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for each pair of agents i and j such that $i \neq j$ there exists a path in g between i and j. A component of a network (N, g), is a nonempty subnetwork (N', g') such that $\emptyset \neq N' \subseteq N$, $g' \subseteq g$ satisfying (i) (N', g') is connected, and (ii) if $i \in N'$ and $ij \in g$, then $j \in N'$ and $ij \in g'$.[5] The set of components of g is denoted by $C(g)$. A component of a network is minimally connected if the path between any two agents in that component is unique. The set of networks composed of minimally connected components is denoted G^m. Formally, $G^m = \{g \in \mathbb{G} \mid \#C(g) < \#C(g - ij) \text{ for each } ij \in g\} \cup \{g^{\emptyset}\}$, where $g^{\emptyset}$ is the empty network. A network is minimally connected if all the agents are in the same minimally connected component. The set of minimally connected networks is $G^M = \{g \in G^m \mid \#C(g) = 1\}$. We use the measure of betweenness centrality of Freeman (1977). Letting $P_i(kj)$ denote the number of shortest paths between k and j that i lies on, and $P(kj) = \Sigma_{i \notin \{k,j\}} P_i(kj)$, the betweenness centrality of an agent i is given by $Ce_i^B(g) = (2/((n-1)(n-2)))\Sigma_{k \neq j : i \notin \{k,j\}} P_i(kj)/P(kj)$. This measure will be used to determine the central agents of a connected line. A connected line is a minimally connected network (N, g) such that no agent in N has more than two links. The set of connected lines is $G^L = \{g \in G^M \mid d_i(g) \leq 2 \text{ for all } i \in N\}$. The central elements of a line $g \in G^L$ are the agents with the highest measure of betweenness centrality. Formally, for $g \in G^L$, $Ce(g) = \{i \in N \mid Ce_i^B(g) \geq Ce_j^B(g)$ for all $j \in N\}$.

Model

We further investigate the model of Bramoullé and Kranton (2007a) where n ex-ante identical individuals are risk averse and face shocks to their income. Each individual's income, y_i, is a random variable which is independently and identically distributed with mean $\overline{y}$ and variance σ^2. Agents have identical preferences, represented by the utility function v, which is increasing and strictly concave in monetary holdings. Individuals may create links with each other. By doing so, they commit to pool their income with the other agents in their component and to share it equally.[6]

[5] This definition of components is proposed by Jackson (2008) and implies that an agent without links in a network is considered as a component.

[6] In reality, full income pooling is not observed. Ligon (1998) finds that information asymmetry is the main factor explaining incomplete income pooling in rural India. Lack of commitment (Coates and Revallion, 1993) is another explanation of this observation. In our model, we assume full information and that agents have the ability to commit to a future contingent transfer. As such,

It follows that risk-sharing benefits only depend on the number of individuals in the component. If agents $1, 2, ..., s$ belong to a component of size s, then the monetary holdings of each agent in this component are $(y_1 + y_2 + ... + y_s)/s$ and their expected utility are given by $u(s) = Ev((y_1 + y_2 + ... + y_s)/s)$, where E denotes the expectation over the realization of incomes. The expected monetary holdings of an agent are independent of the network, but the variance of her expected monetary holdings is decreasing with the size of the component to which she belongs. Since agents are risk-averse, the expected utility function $u(s)$ is increasing in the size of the component, that is $u(s + 1) > u(s)$ for all integer s. In addition, we assume that it increases at a decreasing rate, i.e. $u(s + 2) - u(s + 1) < u(s + 1) - u(s)$ for all s. That is, the bigger is the set of agents with whom an agent shares her risk, the smaller is her benefit to have a new insurance partner. Each direct link ij results in a cost c to both i and j. This cost should be interpreted as an amount of resources needed to ensure that the transfers are realized ex-post, once the shock is realized. In other words, it is assumed that a richer agent will share her revenue with a poorer agent to whom she is linked, because those agents have developed a relationship of trust among themselves, which was costly to establish. We assume that these costs are non-monetary and as such, they cannot be shared with other members of the component. Bramoullé and Kranton (2007a) have motivated this assumption by saying that "some costs, such as the time incurred to build a relation are not easy to compensate or transfer". The payoff of agent i in the network g is given by

$$U_i(g) = u(s_i) - d_i(g)c,$$

where $d_i(g)$ indicates the number of links agent i has and s_i denotes the size of the component to which she belongs, $s_i = \#S$, where $i \in S$ and $(S, h) \in C(g)$.

Efficiency

A network $g \in \mathbb{G}$ is *efficient* if it maximizes the total societal value, that is if $\Sigma_{i \in N} U_i(g) \geq \Sigma_{i \in N} U_i(g')$ for all $g' \in \mathbb{G}$. Efficient networks are composed of minimally connected components since otherwise, productive resources would be wasted. The total utility of the agents in a network g composed of k minimally connected components is given by $\Sigma_{i \in N} U_i(g) = \Sigma_{j=1}^{k} s_j u(s_j) - 2c(n - k)$, where s_j

the choice of the equal sharing rule seems appropriate as it is the optimal one.

is the size of the component j. The assumption on the expected utility function ensures that the total value of a component is increasing with the size of this component, namely $(s+1)u(s+1) > su(s)$. Bramoullé and Kranton (2007a) have shown that this total value could increase at an increasing or decreasing rate, *i.e.* $(s+2)u(s+2) - (s+1)u(s+1)$ could be bigger or smaller than $(s+1)u(s+1) - su(s)$.[7] We assume in this paper that the total value of a component is increasing with the size of this component at a nondecreasing rate. Efficient networks can then only be of two types: either nobody is linked or everybody is indirectly connected. Let us note by $c^* = (n[u(n) - u(1)])/(2(n-1))$ the critical cost of link formation such that the empty network generates the same total utility than a minimally connected network. When $(s+2)u(s+2) - (s+1)u(s+1) > (s+1)u(s+1) - su(s)$ for all $s \in \{1, 2, ..., n-2\}$, the empty network is efficient if $c > c^*$, while an efficient network is composed of one component connecting minimally the n agents if $c < c^*$.

3 Stable risk-sharing networks when agents are myopic

In this section, we investigate the formation of stable risk-sharing networks when agents are myopic. We adopt the notion of pairwise myopically stable sets due to Herings, Mauleon and Vannetelbosch (2009) which is a generalization of Jackson and Wolinsky (1996) pairwise stability notion. A pairwise myopically stable set is such that from any network outside this set, there is a myopic improving path leading to some network in the set, and each deviation outside the set is deterred because the deviating agents do not prefer the resulting network. The notion of myopic improving path is due to Jackson and Watts (2002) and is defined as a sequence of networks that might be observed when agents are adding or deleting links, one at a time, in order to improve their current payoff. Formally, a myopic improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either: (i) $g_{k+1} = g_k - ij$

[7] No general properties of v and y determine the curvature of $su(s)$. Bramoullé and Kranton (2007a) have shown that when the primitive utility function is CARA: $v(y) = v_0 - e^{-\mu y}$, where $\mu > 0$ denotes the level of absolute risk-aversion, and if income is normally distributed, then $su(s)$ is increasing with s at a decreasing rate, while if we consider the quadratic utility function: $v(y) = y - \lambda y^2$, where λ is a positive parameter, then $su(s)$ is increasing linearly with s.

for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ or $U_j(g_{k+1}) > U_j(g_k)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $U_j(g_{k+1}) \geq U_j(g_k)$. For a given network g, we denote by $M(g)$ the set of networks that can be reached through a myopic improving path from g.

Definition 1. A set of networks $G \subseteq \mathbb{G}$ is pairwise myopically stable if

(i) $\forall\, g \in G$,

(ia) $\forall\, ij \notin g$ such that $g+ij \notin G$, $(U_i(g+ij), U_j(g+ij)) = (U_i(g), U_j(g))$ or $U_i(g+ij) < U_i(g)$ or $U_j(g+ij) < U_j(g)$,

(ib) $\forall\, ij \in g$ such that $g - ij \notin G$, $U_i(g-ij) \leq U_i(g)$ and $U_j(g-ij) \leq U_j(g)$,

(ii) $\forall g' \in \mathbb{G} \setminus G$, $M(g') \cap G \neq \emptyset$,

(iii) $\nexists\, G' \subsetneq G$ such that G' satisfies Conditions (ia), (ib), and (ii)..

Conditions (ia) and (ib) in Definition 1 capture deterrence of external deviations. In Condition (ia) the addition of a link ij to a network $g \in G$ that leads to a network outside G is deterred because the two agents involved do not prefer the resulting network to network g. Condition (ib) is a similar requirement, but then for the case where a link is severed. Condition (ii) requires external stability. External stability asks for the existence of a myopic improving path from any network outside G leading to some network in G. Notice that the set $\mathbb{G}$ (trivially) satisfies Conditions (ia), (ib), and (ii) in Definition 1. This motivates Condition (iii), the minimality condition. Jackson and Watts (2002) have defined the notion of a closed cycle. A closed cycle is a set of networks C such that for any pair of networks $g, g' \in C$, where $g \neq g'$, there exists a myopic improving path from g to g', and each myopic improving path emanating from a network in the set C does not reach a network outside C. Each pairwise stable network is a closed cycle. Herings, Mauleon and Vannetelbosch (2009) have proved that the pairwise myopically stable set coincides with the set of networks that belong to a closed cycle.

Bramoullé and Kranton (2007a) have shown that the set of closed cycles consists only of pairwise stable risk-sharing networks if some exist. In addition, they have analyzed the architecture of pairwise stable networks and the conditions on the parameters that guarantee their existence. To summarize their results, let s^* be the critical size of a component such that the benefit of adding another member to the component is less than the cost of doing so:

$$s^* = \max\{s \in \mathbb{N} \mid u(s) - u(s-1) \geq c\}.$$

Since the expected utility function u is increasing at a decreasing rate, an agent belonging to a component of size $s < s^*$ is willing to add a link with a singleton while an agent belonging to a component of size $s > s^*$ has incentives to cut a link if she reaches a component of size $s-1$. Another threshold s^{**} defines the maximal size of a component of a pairwise stable network if another component in the network has size s^*. The threshold s^{**} is defined as follows:

$$s^{**} = \max\{s \leq s^* \mid u(s^* + s) - u(s^*) \leq c, \text{ with the inequality being strict if } s < s^*\}.$$

An agent in a component of size s^* is not willing to add a link with an agent in a component of size smaller than or equal to s^{**}. Each component of a pairwise stable network is minimally connected and at least one of these components has size $s = \min\{s^*, n\}$. If $s^* = s^{**}$, pairwise stable networks always exist. They are composed of a maximal number of components of size s^* and of one component of smaller size. If $s^* > s^{**}$, pairwise stable networks exist if and only if $s^* + s^{**} \geq n$. They are then composed of one component size s^* and of another of size $n - s^*$. Let G^* be the set of networks composed of minimally connected components of size s^* and of one minimally connected component of size $s = n - s^* \operatorname{int}(n/s^*)$ if $s \neq 0$.[8] Formally, $G^* = \{g \in G^m \mid$ if $(S, h) \in C(g)$ and $\#S \neq s^*$, then (i) $\#S < s^*$ and (ii) for all $(S', h') \in C(g)$ with $S' \neq S$, we have $\#S' = s^*\}$. Bramoullé and Kranton (2007a) have shown that the pairwise myopically stable set G is a superset of G^*. Furthermore, the two sets G and G^* coincide if and only if $s^* + s^{**} \geq n$, or $s^* = s^{**}$.

Proposition 1. (Bramoullé and Kranton, 2007) The pairwise myopically stable set G is such that $G^* \subseteq G$. In addition, G^* is the unique pairwise myopically stable set if and only if either $s^* = s^{**}$, or $n \leq s^{**} + s^*$.

All proofs are presented in the appendix. Let us introduce another threshold, $\overline{s}$, which is the maximal integer such that two agents in different components of size $\overline{s}/2$ are willing to add a link between them. Formally, it is defined as follows:

$$\overline{s} = \max\{s \in \mathbb{N} \mid \text{(i) } s \text{ is even and (ii) } u(s) - u(s/2) > c\}.$$

[8] The operator int(x) gives the integer part of the real x.

Notice that the addition of a link is not profitable for at least one of the agents involved if one of them belongs to a component of size bigger than $\overline{s}/2$.[9] The following proposition states that each network in the pairwise myopically stable set is composed of minimally connected components of size smaller than or equal to $\overline{s}$, and contains at least $s^* - 1$ links, but no more than $n - 1 - \text{int}((n-1)/\overline{s})$ links.

Proposition 2. Each network g in the pairwise myopically stable set G is such that (i) $g \in G^m$, (ii) $\#S \leq \overline{s}$ for all $(S, h) \in g$ and (iii) $s^* - 1 \leq d(g) \leq n - 1 - \text{int}((n-1)/\overline{s})$.

Intuitively, there exists a myopic improving path from every network composed of components which are not minimally connected to some network composed of minimally connected components if the agents delete unnecessary links. However, the converse does not hold. Once a network composed of minimally connected components is reached, every myopic improving path leads to other networks in G^m since the addition of a useless link is costly. In addition, from networks composed of big-sized components, agents are willing to cut links with peripheral agents (agents having only one link) as long as the size of their component is bigger than s^*. On the other hand, from networks composed of small-sized components, no myopic improving path is leading to a network having a component of size bigger than $\overline{s}$ since, if it was the case, an agent should add a link at some point in the path when she is currently a member of a component of size $s > \overline{s}/2$, but the addition of that link is not profitable. Each network of the pairwise myopically stable set has more than $s^* - 1$ links since no agent is willing to cut a link if there are s^* agents or fewer in her component. Finally, each network in G has less than $n - 1 - \text{int}((n-1)/\overline{s})$ links as this number of links is obtained when the agents form a maximal number of components of size $\overline{s}$.

4 Stable risk-sharing networks when agents are farsighted

Myopic agents are assessing the profitability of their decision to create new mutual insurance agreements or to remove old ones by considering that their choice has no impact on others' decisions. In this section, we analyze the formation risk-sharing

[9] This result is established in the appendix (see Lemma 3.A.2).

networks when agents are farsighted, rather than myopic, in the sense that they are able to anticipate how other agents would react to their choice of partners.

Herings, Mauleon and Vannetelbosch (2009) have proposed a solution concept to address the question of stability when agents are farsighted: the pairwise farsightedly stable set. Before defining the concept, let us introduce the notion of a farsighted improving path, which is the counterpart of the myopic improving path described in the previous section. A *farsighted improving path* is a sequence of networks that can emerge when agents form or sever links based on the improvement the end network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two agents involved must both prefer the end network to the current network, with at least one of the two strictly preferring the end network. If a link is deleted, then it must be that at least one of the two agents involved in the link strictly prefers the end network. Formally, it is defined as follows. A farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either: (i) $g_{k+1} = g_k - ij$ for some ij such that $U_i(g_K) > U_i(g_k)$ or $U_j(g_K) > U_j(g_k)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $U_i(g_K) > U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$. For a given network g, let $F(g) = \{g' \in G \mid$ there is a farsighted improving path from g to $g'\}$.

We now introduce the concept of pairwise farsightedly stable set. It is a set of networks such that (i) the deletion or addition of any link from a network in the set leading to a network outside the set is deterred by a credible threat of ending worse off, once other agents further react to the initial deviation, (ii) from any network outside the set, there is a farsighted improving path leading to some network in the set, and (iii) no proper subset of this set satisfies the two first conditions. Formally, pairwise farsightedly stable sets are defined as follows.

Definition.2. A set of networks $G \subseteq \mathbb{G}$ is pairwise farsightedly stable with respect v and Y if

(i) $\forall\, g \in G$,

(ia) $\forall\, ij \notin g$ such that $g + ij \notin G$, $\exists\, g' \in F(g+ij) \cap G$ such that $(Y_i(g', v), Y_j(g', v)) = (Y_i(g, v), Y_j(g, v))$ or $Y_i(g', v) < Y_i(g, v)$ or $Y_j(g', v) < Y_j(g, v)$,

(ib) $\forall\, ij \in g$ such that $g - ij \notin G$, $\exists\, g', g'' \in F(g - ij) \cap G$ such that $Y_i(g', v) \leq Y_i(g, v)$ and $Y_j(g'', v) \leq Y_j(g, v)$,

(ii) $\forall g' \in \mathbb{G} \setminus G$, $F(g') \cap G \neq \varnothing$.

(iii) $\nexists\, G' \subsetneq G$ such that G' satisfies Conditions (ia), (ib), and (ii).

Condition (i) in Definition 2 requires the deterrence of external deviations. Condition (ia) captures that adding a link ij to a network $g \in G$ that leads to a network outside of G, is deterred by the threat of ending in g'. Here g' is such that there is a farsighted improving path from $g + ij$ to g'. Moreover, g' belongs to G, which makes g' a credible threat. Condition (ib) is a similar requirement, but then for the case where a link is severed. Condition (ii) in Definition 2 requires external stability and implies that the networks within the set are robust to perturbations. From any network outside of G there is a farsighted improving path leading to some network in G. Notice that the set $\mathbb{G}$ (trivially) satisfies Conditions (ia), (ib), and (ii) in Definition 2. This motivates the requirement of a minimality condition, namely Condition (iii).

We now provide a partial characterization of the pairwise farsightedly stable sets of risk-sharing networks. We analyze the case of very small costs of link formation, the case of small costs of link formation and the case of high costs of link formation.

Very small costs of link formation

Proposition 3 characterizes partially the pairwise farsightedly stable sets when the costs of link formation satisfy $c < u(n) - u(n-1)$. For such costs, we find that (a) each pairwise farsightedly stable set contains at least one connected network, (b) each set G composed of a minimally connected network $\widetilde{g} \in G^M$ and all other networks $g \in G^M$ such that $U_i(g) = U_i(\widetilde{g})$ for all $i \in N$ is a pairwise farsightedly stable set, and (c) each set G composed of a minimally connected network $g_1 \in G^M$ and another minimally connected network $g_2 \in G^M$ such that $U_i(g_1) \neq U_i(g_2)$ for some agent $i \in N$ and g_1 and g_2 are not star networks (i.e. they are not such that one agent connects directly all the others) is a pairwise farsightedly stable set.

Proposition 3. If $0 < c \leq u(n) - u(n-1)$, then

(a) If G is a pairwise farsightedly stable set, then $\#C(g) = 1$ for some $g \in G$.

(b) For each $\widetilde{g} \in G^M$, the set $G(\widetilde{g}) = \{g \in G^M \mid U_i(g) = U_i(\widetilde{g}) \text{ for all } i \in N\}$ is a pairwise farsightedly stable set.

(c) The set $G = \{g_1, g_2\} \subseteq G^M$ such that $U_i(g_1) \neq U_i(g_2)$ for some agent $i \in N$ and both g_1 and g_2 are not star networks is a pairwise farsightedly stable set.

A connected line Pareto dominates each network composed of multiple components for such costs. There are thus no farsighted improving paths from a connected line to a network composed of multiple components. It then follows that a set of networks that does not include at least one network that connects indirectly all the agents is not externally stable. To prove part (b) and (c), we first show that there is a farsighted improving path from any network outside $G(\widetilde{g}) = \{g \in G^M \mid U_i(g) = U_i(\widetilde{g})$ for all $i \in N$, for some $\widetilde{g} \in G^M\}$ leading to each network in $G(\widetilde{g})$. Intuitively, this holds since from any network g outside $G(\widetilde{g})$, there always exists an agent willing to cut a link from g or a pair of agents willing to add a link from g, looking forward to the formation of a network in $G(\widetilde{g})$. Thus, the sets proposed in part (b) and (c) of the proposition are such that there are farsighted improving paths from each network outside the set to some network in the set. Notice in addition that each pairwise deviation from a network in one of those sets is deterred by the threat of coming back at the same network in one step.

A star network as a singleton is a pairwise farsightedly stable set according to part (b) of Proposition 3. It is thus required that g_1 and g_2 are not star networks in part (c) of Proposition 3 as otherwise, the set of networks g_1 and g_2 would fail to be minimal. In the last section of the paper, we analyze the quadratic utility function case and we show that when there are four agents, some pairwise farsightedly stable sets of networks are exclusively composed of networks connecting all the population but not at the minimal cost. Whether this result holds for general utility function and for any number of agents remains an open question.

Small costs of link formation

In the next proposition, we show that each set composed of connected line $\widetilde{g}$ and of all other lines where the payoff of the agents is equal to their payoff in $\widetilde{g}$ constitutes a pairwise farsightedly stable set if the cost of link formation satisfies $c < \min\{u(n) - u(\text{int}((n+1)/2)), (u(n) - u(1))/2\}$.

Proposition 4. If $c < \min\{u(n) - u(\text{int}((n+1)/2)),\ (u(n) - u(1))/2\}$, we have that

(a) For each $\widetilde{g} \in G^L$, the set $G(\widetilde{g}) = \{g \in G^L \mid U_i(g) = U_i(\widetilde{g})$ for all $i \in N\}$ is a pairwise farsightedly stable set.

(b) The set $\{g_1, g_2\}$ where $g_1, g_2 \in G^L$ and $U_i(g_1) \neq U_i(g_2)$ for some $i \in N$ is a pairwise farsightedly stable set.

In the proof of this proposition, it is first established that there is a farsighted improving path from any network outside $G(\widetilde{g}) = \{g \in G^L \mid U_i(g) = U_i(\widetilde{g})$ for all $i \in N$, for some $\widetilde{g} \in G^L\}$ leading to each network in $G(\widetilde{g})$. From a network outside $G(\widetilde{g})$, farsighted agents who have more links than in the connected line $\widetilde{g}$ or who have the same number of links but are indirectly connected to less than n agents, cut their links until the empty network is reached, looking forward to the formation of the network $\widetilde{g}$. From the empty network, the agents add links in order to build $\widetilde{g}$ such that the last link to be added is the central link of the line. This last move is profitable for the two agents involved in that link since $c < u(n) - u(\text{int}((n+1)/2))$. The addition of each other link from the empty network to $\widetilde{g}$ is profitable for farsighted agents having already one link as this allows them to move from a component of size smaller than $\text{int}((n+1)/2)$ to the connected network $\widetilde{g}$, and it is profitable for isolated agents since $u(n) - 2c > u(1)$. Notice in addition that each pairwise deviation from a network in the set is deterred by the threat of coming back at the same network.

The pairwise farsightedly stable sets described in Proposition 4 are not necessarily unique. We will see in Section 5 that inefficient networks may also belong to some pairwise farsightedly stable sets.

High costs of link formation

In Proposition 5, we show that when the cost of link formation is sufficiently high, (a) the set of all networks in which each agent belongs to a component of size 2 is the only pairwise farsightedly stable set if the number of agent is even, (b) the set of all networks in which the same agent is not connected while the remaining agents belong to a component of size 2 is a pairwise farsightedly stable set if the number of agent is odd, and (c) the set composed of one network where a maximal number of linked pairs forms among all the agents but agent k and of one network where a maximal number of linked pairs forms among all the agents but agent $l \neq k$ is a pairwise farsightedly stable set if the number of agent is odd.

Proposition 5. If $u(n) - u(2) < c < u(2) - u(1)$, then

(a) The set $G = \{g \in \mathbb{G} \mid d_i(g) = 1$ for all $i \in N\}$ is the unique pairwise farsightedly stable set if n is even.

(b) For $k \in N$, the set $G_k = \{g \in \mathbb{G} \mid d_i(g) = 1$ for all $i \in N \setminus \{k\}$, and $d_k(g) = 0\}$ is a pairwise farsightedly stable set if $(u(3) - u(1))/2 < c$ and n is odd,

(c) For $k, l \in N$ with $k \neq l$, the set $G = \{g_k, g_l\}$, where for $m \in \{k, l\}$, $G_m = \{g \in \mathbb{G} \mid d_i(g) = 1$ for all $i \in N \setminus \{m\}$, and $d_m(g) = 0\}$ is a pairwise farsightedly stable set if $(u(3) - u(1))/2 < c$, n is odd and $n \geq 5$.

When the costs of link formation satisfy $u(n) - u(2) < c < u(2) - u(1)$, each agent prefers to be in a network in which she is a member of a component of size two rather than in any network in which her number of links is different than one. From any network that does not belong to the set of networks composed of a maximal number of linked pairs, agents having more than one link are willing to cut a link, and agents having no links are willing to add a link, looking forward to a network composed of a maximal number of linked pairs. Part (a) of the proposition is then derived from the fact that there are no farsighted improving paths emanating from a network composed of a maximal number of linked pairs if n is even. When the number of agents is odd, an agent, say k, is not connected in a network composed of a maximal number of linked pairs $g \in G_k$. Then, part (b) and part (c) of the proposition follow from the fact that from each network outside G_k, there is a farsighted improving path to each network in G_k. Also, each deviation from G_k is deterred by the threat of coming back at the same network in one step.

The characterization of pairwise farsightedly stable sets when the cost of link formation is intermediate, i.e. when $\min\{(u(n) - u(1))/2, u(n) - u(n/2)\} \leq c \leq u(n) - u(2)$ for n even, or when $\min\{(u(n) - u(1))/2, u(n) - u((n+1)/2)\} \leq c \leq \max\{(u(3) - u(1))/2, u(n) - u(2)\}$ for n odd, remains an open question.

Let us summarize the results obtained concerning the structure of risk-sharing networks formed by farsighted agents and compare them with the networks formed by myopic agents and the efficient ones. For very small costs of link formation ($c \leq u(n) - u(n-1)$) or high ones ($u(n) - u(2) < c$ when the population size is even, or $\max\{u(n) - u(2), (u(3) - u(1))/2\} < c$ if the population size is odd), farsighted and myopic agents form the same networks. For very small costs of link formation, the pairwise myopically stable set is the set of efficient networks (each efficient network is pairwise stable since $s^* \geq n$ for such costs), and each efficient network belongs to some pairwise farsightedly stable set. For high costs of link formation, the pairwise myopically stable set contains all networks composed of a maximal number of linked pairs ($s^* = 2 = s^{**}$ when $u(n) - u(2) < c < u(2) - u(1)$, implying that each network in the pairwise myopically stable set is pairwise stable). The union of the pairwise farsightedly stable sets coincides with the pairwise myopically stable set when n is even, and contains the pairwise myopically stable set when n is odd. Whether a network not contained in the pairwise myopically stable set belongs to some pairwise farsightedly stable set when n is odd remains an open question. The networks formed differ however for small costs of link formation. When $u(n) - u(n-1) < c < \min\{u(n) - u(\text{int}((n+1)/2)), (u(n) - u(1))/2\}$, each line connecting all the agents and other lines in which the degree of the agents is similar constitutes a pairwise farsightedly stable set. Farsighted agents may form efficient networks, while myopic agents cannot sustain those networks at equilibrium since from an efficient network, the agents have myopic incentives to cut a link with a peripheral agent as long as $u(n) - u(n-1) < c$. Farsighted agents are not willing to cut those links as they fear that the others will in turn modify sequentially their choice of insurance partners so that the network that will form in the end will be the same line connecting all the agents.

5 Application: the quadratic utility function

In this section, we analyze more in detail the formation of risk-sharing networks when agents have a quadratic utility function. We illustrate through this example the implications of the theorems presented in the previous sections about the formation of risk-sharing networks by farsighted and myopic agents. In particular, we investigate three questions. First, we analyze what is the impact of the risk-aversion of the agents, of their initial wealth, and of the variance of the income shock on the formation of risk-sharing networks. Second, we study to which extent the range of costs for which farsightedly stable networks can be identified shrinks as the size of the population increases. Third, we investigate whether additional results can be obtained when the characterization we have established is incomplete, that is when the cost of link formation is very small, small or intermediate.

Let v be a quadratic utility function $v(y) = y - \lambda y^2$ where λ represents the level of risk-aversion of an individual. As shown in Bramoullé and Kranton (2007a), the expected utility function is then $u(s) = v(\overline{y}) - (\lambda\sigma^2)/s$ where $\overline{y}$ and σ^2 are respectively the mean and the variance of the income distribution. This expected utility function is increasing at a nondecreasing rate, and the total utility of the members of a component ($su(s)$) is increasing at a constant rate, so that quadratic utility functions verify our assumptions.

The stability of sets of networks is determined by comparing the expected utility of an agent when she belongs to components of different sizes. For quadratic utility functions, the variation of expected utility of an agent if she moves from a component of size k to a component of size l is $u(l) - u(k) = \lambda\sigma^2(l-k)/lk$. As far as stability is concerned, the mean of income ($\overline{y}$) thus does not matter. The relevant parameters of the utility functions are the variance of the shock (σ^2) and the parameter that represents the risk aversion (λ). In addition, only their product matters so that uncertainty and risk aversion play the same role.

We have depicted in Figure 1 the evolution of the thresholds of link cost that are relevant for the theorems as a function of the number of agents.[10] The uncertainty (σ^2) and the risk aversion (λ) change the scale of Figure 1 but not its shape since a modification of one of those parameters affects the various thresholds in the same way. We assume in Figure 1 and in the rest of this section that $\lambda\sigma^2 = 9$. When the number of agents increases, the range of costs for which Propositions 3.3, 3.4 and 3.5 applies shrinks while the range of intermediate costs (i.e. for which we have not characterized the pairwise farsightedly stable sets) increases. For high costs of link formation ($u(n) - u(2) < c < u(2) - u(1)$ when n is even and $\max\{u(n) - u(2), (u(3) - u(1))/2\} < c < u(2) - u(1)$ when n is odd), the lower bound of the interval is nondecreasing with n while the upper bound is fixed. For very small costs of link formation ($0 < c \leq u(n) - u(n-1)$), the upper bound of the interval is decreasing with n since by assumption, the bigger is the set of agents with whom an agent shares her risk, the smaller is her benefit to have a new insurance partner. When n is even, the range of costs for which Proposition 4 applies ($u(n) - u(n-1) < c < u(n) - u(\text{int}(n+1)/2)$) decreases with n as long as $n \geq 4$. Similarly, this range decreases with n when n is odd for $n \geq 5$. The range of intermediate costs is determined by the condition $u(n) - u(n/2) \leq c \leq u(n) - u(2)$ when n is even and by the condition $u(n) - u(\text{int}((n+1)/2)) \leq c \leq \max\{u(n) - u(2), (u(3) - u(1))/2\}$ when n is odd. It is increasing with n since the lower bound of the interval ($u(n) - u(\text{int}((n+1)/2))$) is decreasing in n while the upper bound is nondecreasing in n.

[10] We have not represented the threshold $(u(n) - u(1))/2$ since $\min\{(u(n) - u(1))/2; u(n) - u(\text{int}((n+1)/2))\} = u(n) - u(\text{int}((n+1)/2))$ when the utility function is quadratic.

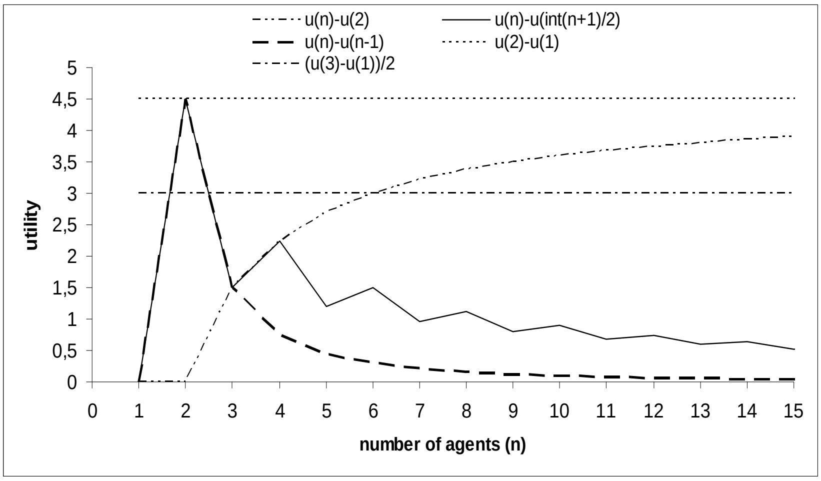

Figure 1. Cost thresholds and population size

The characterizations proposed in the theorems are not complete. The theorems identify some equilibrium candidates but there may exist other equilibria. We have developed an algorithm that aims at identifying all the pairwise farsightedly stable sets to investigate whether additional results can be obtained when the characterization we have established is incomplete.[11] Among n agents, there are possibly $K = \Sigma_{i=0}^{n(n-1)/2} C_i^{n(n-1)/2}$ networks and $\Sigma_{i=1}^{K} C_i^K$ equilibrium set candidates.[12] It is thus increasingly complex to fully characterize the pairwise farsightedly stable sets since the number of candidates explodes as n increases. When $n = 3$, there are 8 different networks that can form 256 different equilibrium set candidates and for $n = 4$, there are 64 different possible networks and $1,8447\ E + 19$ candidates.[13] Having associated to each network a number between 1 and K, the output of the algorithm is (i) a square matrix F of dimension $K \times K$, where K is the total number of networks among n agents, such that $F(i,j) = 1$ if there is a farsighted improving path from the network number $i \in \{1, K\}$ leading to the network $j \in \{1, K\}$ and (ii) a matrix $PFFS$ of dimension $L \times K$, where L is the total number of pairwise farsightedly stable sets such that a set composed of networks associated with non-zero elements of a line is a pairwise farsightedly stable set of networks.

We have used the algorithm to determine the farsightedly stable sets of networks formed among 3 agents. For more than 3 agents, we are not able to build a matrix

[11]The algorithm is available upon request from the author. We explain in Appendix 3.B the main steps for its construction.

[12]C_k^n gives the combination of n things taken k at a time without repetition and is equal to $n!/(k!(n-k)!)$.

[13]When $n = 5$, there are 1024 different networks and an infinite number of candidates.

whose number of lines corresponds to the number of candidates. We then have considered subsets of the full set of equilibrium candidates.

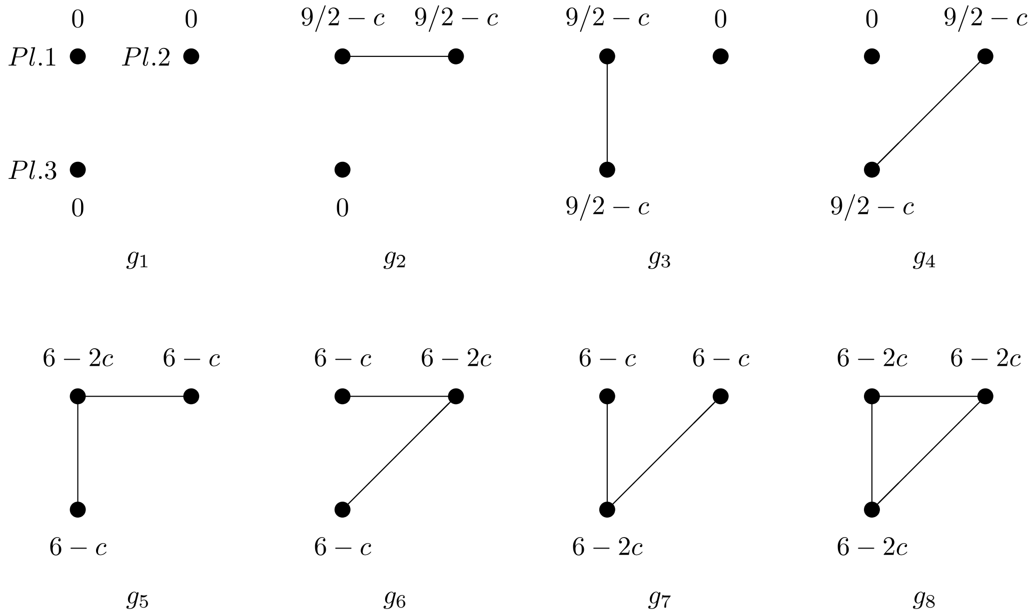

Figure 2. Risk-sharing networks among 3 agents.

In Figure 2, we depict the risk-sharing networks that could be formed among three agents. We assume that $\lambda\sigma^2 = 9 = v(\overline{y})$ so that the payoff of the agents is normalized to 0 in the empty network. Proposition 1 completely characterizes the pairwise myopically stable set of networks which coincides with the set of pairwise stable networks. The simulations reveal that when $c \leq 1,5$ and $c > 3$ there are no other pairwise farsightedly stable sets than those described in Theorems 3.4 and 3.5. When $1,5 < c \leq 3$, a set composed of two networks connecting 2 agents is a pairwise farsightedly stable set, and a set composed of one network connecting 2 agents and a star network, where the hub in the star is not connected in the linked pair network is a pairwise farsightedly stable set of networks.[14] Farsighted agents thus form efficient networks while myopic agents do not. Table 1 summarizes these results.

[14] The hub in a star is the agent who is directly connected to all the other agents.

Cost	Pairwise farsightedly stable set	Pairwise myopically stable set
$0 < c \leq 1,5$	$\{g_5\}$,$\{g_6\}$,$\{g_7\}$	$\{g_5, g_6, g_7\}$
$1,5 < c \leq 3$	$\{g_2, g_3\}$,$\{g_2, g_4\}$,$\{g_3, g_4\}$, $\{g_2, g_7\}$,$\{g_3, g_6\}$,$\{g_4, g_5\}$	$\{g_2, g_3, g_4\}$
$3 < c < 4,5$	$\{g_2\}$,$\{g_3\}$,$\{g_4\}$	$\{g_2, g_3, g_4\}$
$c = 4,5$	$\{g_1, g_2, g_3, g_4\}$	$\{g_1, g_2, g_3, g_4\}$
$c > 4,5$	$\{g_1\}$	$\{g_1\}$

Table 1. Farsightedly and myopically stable sets when n=3 and $\lambda\sigma^2$=9.

In Figure 3, we have depicted all the networks that could be formed among 4 agents. Table 2 summarizes the results obtained with the simulations. To simplify the presentation, we have not written down all the equilibrium candidates, but rather all the classes of equilibrium candidates. The candidates that are symmetric to those identified in Table 2 are also farsightedly stable. By Proposition 5, the only pairwise farsightedly stable set is the set of all linked pairs networks when $2,25 < c < 4,5$. When $c < 2,25$, each set composed of two lines of 4 agents is a pairwise farsightedly stable set (Proposition 4). Also, when $c \leq 0,75$, each star network as a singleton is a pairwise farsightedly stable set (Proposition 3). These propositions cover all the costs but $c = 2,25$, and as long as $c < 2,25$, the characterization may be incomplete. The simulations we have realized do not allow us to provide a complete identification of the pairwise farsightedly stable sets. We have however considered all candidates of at most six networks for all costs of link formation. We have also considered sets of more than six networks, by focusing our attention on specific candidates.[15]

[15]For instance, we have considered sets of ten networks by eliminating the candidates involving networks that are not minimally connected. By doing so, we have identified pairwise farsightedly stable sets of more than six networks when $1,5 < c \leq 2,25$.

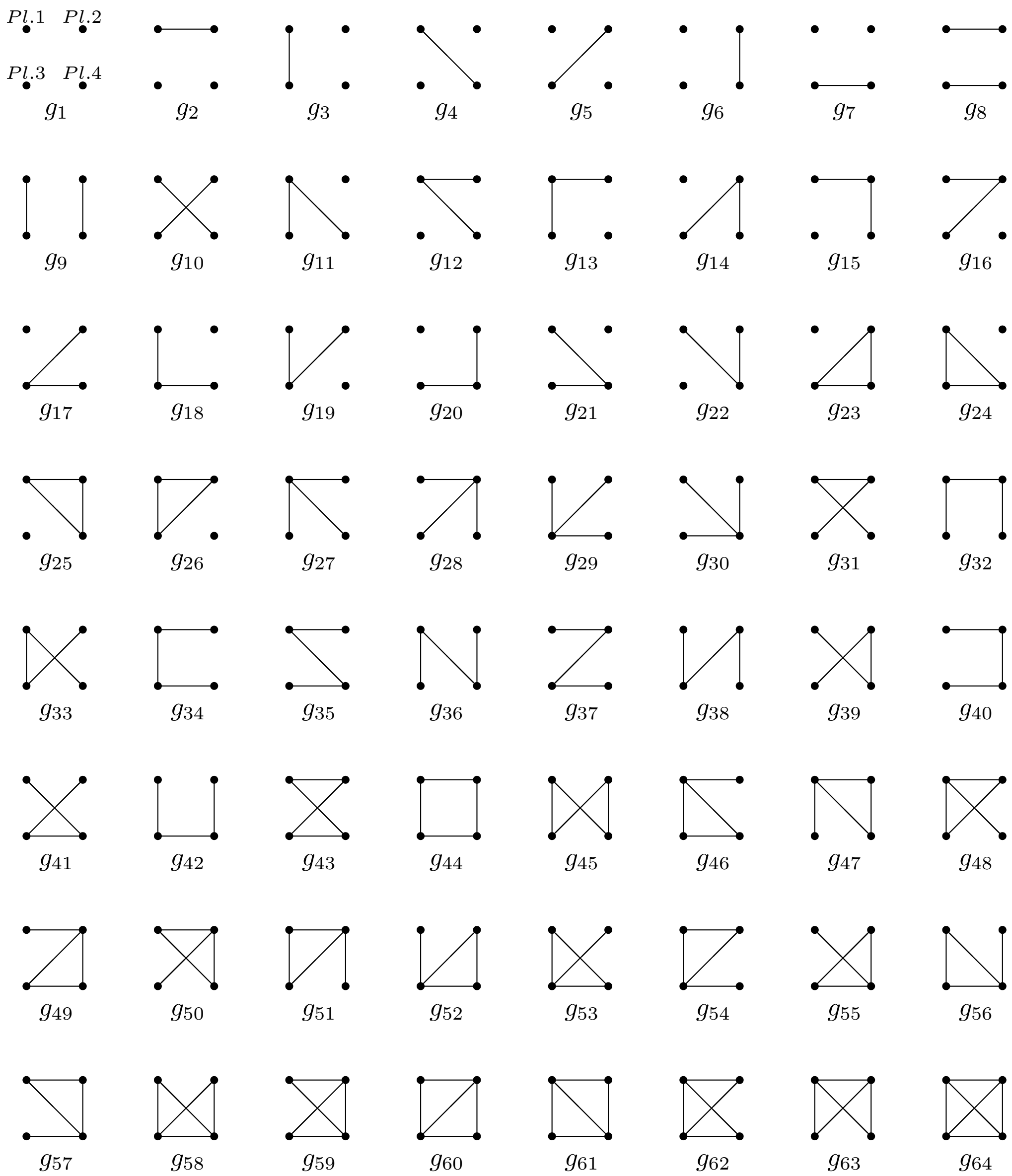

Figure 3: Risk-sharing networks among four agents

When $c = 2,25$, there are no farsighted improving paths from the networks composed of a maximal number of linked pairs, implying that they belong to each pairwise farsightedly stable sets of networks. The set G of all networks composed of a maximal number of linked pairs and of the lines of 4 agents is a pairwise farsightedly stable set. Indeed, there is a farsighted improving path from each network other than the lines of 4 agents or the circles[16] of 4 agents leading to each network composed

[16] A circle is a network where each agent has two links.

of a maximal number of linked pairs. The set G thus satisfies external stability. The deviations from the networks composed of a maximal number of linked pairs leading to a network outside the set necessarily involve the deletion of a link. These deviations are deterred by the threat of coming back at the same network in one step (see Lemma A.3). Deviations from lines of 4 agents involving the addition of a link are deterred by application of Lemma A.3, while those involving the deletion of a link are deterred by the threat of ending in a network composed of a maximal number of linked pairs. Minimality is satisfied since any subset of G would violate external stability. A set G composed of the three networks involving a maximal number of linked pairs, two networks of one link such that each agent has a total of one link in those two networks, and three lines of 3 agents (g_a, g_b, g_c) such that each agent has a total of three links in those three networks, is also a pairwise farsightedly stable set. We have $g \in F(g')$ for some $g \in \{g_a, g_b, g_c\}$, for all $g' \in G^L \cup G^c$, where G^c is the set of circles of 4 agents. External stability is thus guaranteed since there is a path from any other network not in the set leading to a network composed of a maximal number of linked pairs. The addition of a link from a network in the set leading to a network outside the set is never profitable by application of Lemma A.3. For the same reason, a deviation involving the deletion of one link from a component of size 2 is deterred. Also, the hub of a line of 3 agents has no incentives to delete one of her links because she may fear to end up without connections in a network of one link. Minimality holds since external stability would be violated for any $G' \subset G$ such that less than three lines of 3 agents belong to G' while deterrence of external deviations would be violated for any $G' \subset G$ such that less than two lines of 2 agents belong to G'. The class of candidates we have just identified remains farsightedly stable when $1,5 < c < 2,25$. However, the set composed of all the lines of 4 agents and all the networks composed of a maximal number of linked pairs is not since it fails to satisfy minimality.[17]

For $0,75 < c \leq 1,5$, a set composed of two lines of 3 agents such that the isolated agent is not the same agent in the two lines and the set of peripheral agents is different in the two networks is a pairwise farsightedly stable set. This candidate trivially satisfies external stability and minimality, and external deviations are deterred by application of Lemma A.3. If the two lines of 3 agents shared the

[17] Any set composed of two lines of 4 agents is a pairwise farsightedly stable set by Proposition 4.

same set of peripheral agents, external stability would be violated since there are no farsighted improving paths from a line of 4 agents leading to a line of 3 agents such that the peripheral agents are identical in those two networks. External stability would also be violated if the isolated agent was the same agent in the two lines of 3 agents because there are no farsighted improving paths from a star leading to a line of 3 agents such that the hub in the star is not connected in the line. It follows that a set composed of three networks, two lines of 3 agents such that the isolated agent is the same agent in those lines and a star where the hub in the star is not connected in the lines, is also a pairwise farsightedly stable set. When $c > 1,5$, those candidates fail to satisfy external stability because there are no farsighted improving paths from a network composed of components of size 2 or less leading to a line of 3 agents. The simulations reveal that another class of candidates is pairwise farsightedly stable when $0,75 < c \leq 1,5$. A set composed of one line of 4 agents and another line of 3 agents such that the set of peripheral agents in the two lines is not identical is a pairwise farsightedly stable set. External stability holds by application of Lemma A.5, which establishes that there is a farsighted improving path from every network leading to a line of 4 agents provided the payoff of the agents is different in the initial and final networks. The deletion of a link from a line of 4 agents is deterred by the threat of coming back to the same network (see Lemma A.5) while every other deviation is deterred by application of Lemma A.3. When $c > 1,5$, such candidates are no longer pairwise farsightedly stable because the hub in the line of 3 agents cannot be deterred from cutting one link. Indeed, the only stable outcome she might reach by doing so is a line of four agents. When $c < 0,75$, a set composed of six networks that are not minimally connected, three networks where one agent has three links while 2 other agents are connected and three networks where another agent has three links while 2 other agents are connected, is a pairwise farsightedly stable set. To see that such candidates satisfy external stability, notice that from a line of 4 agents, there is always an agent i with two links in the line who may cut successively her two links, looking forward to the following succession of moves: one agent deletes the remaining link, then the 3 agents other than agent i add a link between them, starting first with the links involving the agent j who has three links in the final network.[18] Finally, agents i and j form a link. Deviations from a

[18] The two agents already connected to agent j add a link between them in this step. This move is profitable for them as long as $c < 0,75$. When $c \geq 0,75$, there are no farsighted improving paths

network in the set are deterred: an agent who cuts a link may always fear to come back in the set with at least the same number of links, and a pair of agents who add a link between them are worse off by doing so, and could come back to the same network in one step by cutting the link they have just added.

Cost	Pairwise farsightedly stable sets	Pairwise myopically stable sets
$0 < c < 0,75$	$\{g_{27}\}$, $\{g_{28}\}$, $\{g_{29}\}$, $\{g_{30}\}$; $\{g_{31}, g_{32}\}$,$\{g_{31}, g_{33}\}$,...,$\{g_{31}, g_{42}\}$; $\{g_{46}, g_{47}, g_{48}, g_{49}, g_{50}, g_{51}\}$	$\{g_{27}, g_{28}, ..., g_{42}\}$
$c = 0,75$	$\{g_{27}\}$, $\{g_{28}\}$, $\{g_{29}\}$, $\{g_{30}\}$; $\{g_{31}, g_{32}\}$,$\{g_{31}, g_{33}\}$,...,$\{g_{31}, g_{42}\}$	$\{g_{27}, g_{28}, ..., g_{42}\}$
$0,75 < c \leq 1,5$	$\{g_{11}, g_{12}\}$,$\{g_{11}, g_{13}\}$,$\{g_{11}, g_{15}\}$, $\{g_{11}, g_{16}\}$,$\{g_{11}, g_{17}\}$,$\{g_{11}, g_{19}\}$, $\{g_{11}, g_{20}\}$,$\{g_{11}, g_{22}\}$; $\{g_{11}, g_{33}\}$,$\{g_{11}, g_{34}\}$,...,$\{g_{11}, g_{42}\}$; $\{g_{31}, g_{32}\}$,$\{g_{31}, g_{33}\}$,...,$\{g_{31}, g_{42}\}$; $\{g_{11}, g_{18}, g_{28}\}$,$\{g_{11}, g_{21}, g_{28}\}$	$\{g_{11}, g_{12}, ..., g_{22}\}$
$1,5 < c < 2,25$	$\{g_{31}, g_{32}\}$,$\{g_{31}, g_{33}\}$,...,$\{g_{31}, g_{42}\}$; $\{g_2, g_7, g_8, g_9, g_{10}, g_{12}, g_{14}, g_{18}\}$	$\{g_2, g_3, ..., g_{22}, g_{31}, g_{32}, ..., g_{42}\}$
$c = 2,25$	$\{g_8, g_9, g_{10}, g_{31}, g_{32}, ..., g_{42}\}$; $\{g_2, g_7, g_8, g_9, g_{10}, g_{12}, g_{14}, g_{18}\}$	$\{g_8, g_9, g_{10}\}$
$2,25 < c < 4,5$	$\{g_8, g_9, g_{10}\}$	$\{g_8, g_9, g_{10}\}$

Table 2. Farsightedly and myopically stable sets when n=4 and $\lambda\sigma^2$=9.

It is not easy to determine whether farsightedness helps reducing a conflict between myopic stability and efficiency, mainly because it implies a comparison of the efficiency of sets of networks. In addition, the pairwise myopically stable set is unique by definition while pairwise farsightedly stable sets are not. In what follows, we summarize the new insights obtained with the simulations when four agents form a risk-sharing network. To compare the same object when discussing the issue of stability versus efficiency, we contrast the set of networks that belong to some pairwise farsightedly stable set with the pairwise myopically stable set. (i)

from a network composed of minimally connected components leading to a network composed of components that are not minimally connected.

For very small costs of link formation ($c \leq 0,75$), myopic agents always form efficient networks. Each efficient network also belongs to some pairwise farsightedly stable but it is not excluded that an equilibrium candidate is composed of networks that are not minimally connected when agents are farsighted, because farsighted agents can move from an efficient network looking forward to an inefficient network where her situation has improved. (ii) When the costs of link formation are small ($0,75 < c < 2,25$), we should further distinguish two cost thresholds. (ii.1) When $c \leq 1,5$, the pairwise myopically stable set consists of all lines of three agents, which are pairwise stable. The set of networks included in some farsightedly stable sets are all the lines of three and four agents, and the star networks. This set is thus the union of the pairwise myopically stable set and the set of efficient networks. (ii.2) When $c > 1,5$, the set of networks included in some pairwise farsightedly stable set and those included in the pairwise myopically stable set consist in all the networks composed of one link, two links, and the lines of four agents. (iii) For intermediate costs of link formation ($c = 2,25$), the pairwise stable networks are composed of a maximal number of linked pairs. Those network also belong to some pairwise farsightedly stable set in addition to the networks composed of lines involving two, three, and four agents (iv) for high costs of link formation ($c > 2,25$), Proposition 5 characterizes completely the farsightedly stable set of networks. It is the set of all the networks composed of a maximal number of linked pairs, and coincides with the pairwise myopically stable set.

In some cases, the set of networks belonging to some pairwise farsightedly stable set coincides with the pairwise myopically stable set. In others, the two sets are different. When the two sets are different, the pairwise myopically stable set is included in the set of networks belonging to some pairwise farsightedly stable set, and each pair of networks in the pairwise myopically stable set generates the same value. The networks that are farsightedly stable but not myopically stable do not necessarily generate more value than the networks that are pairwise stable. This is indeed the case when the costs of link formation are very small or intermediate. When the costs are small on the other hand, each network that is farsightedly stable but not myopically stable is efficient.

6 Conclusion

In this paper, we have analyzed the formation of risk-sharing networks. A growing empirical literature (see Fafchamps, 1992; Grimmard, 1997; Fafchamps and Lund, 2003; De Weerdt and Dercon, 2006) has shown that a fully efficient risk-pooling equilibrium is not reached: risk-sharing does not take place within exogenous group such as the village, but rather within networks involving agents having common characteristics (neighborhood, professional or religious affiliation, kinship, etc). Most of the theoretical papers on informal risk-sharing in developing countries assume that no binding agreement can be enforced (see Kimball (1988); Coate and Revallion (1993); Kocherlakota (1996); Ligon, Thomas and Worrall (2002); Genicot and Ray (2003); Bloch, Genicot and Ray (2005). These papers model the risk-sharing process as a dynamic game where agents have incentives to transfer money today because they expect to be reciprocated at a future date. The self-enforcing transfer schemes identified involve incomplete income pooling, providing a theoretical argument to explain the observed pattern of informal insurance relationships.

Platteau (2000) has argued that kinship groups, membership of a clan or of a religious group are factors that help to imposing norms on members, enhancing trust and that increase the ability to punish deviant behaviors, thereby making risk-sharing easier. Thus, the lack of formal institutions allowing agents to commit to future transfers may be relevant at the village level, but not within the aforementioned communities.

Bramoullé and Kranton (2007a) have developed a model of insurance network formation, where agents invest in costly bilateral relationships in order to become members of a group of agents insuring each other against income or expenditure shocks. Their model is a decentralized model of coalition formation, where a coalition is a set of agents that are directly or indirectly connected to each other. Each member of a coalition commits to share her income with her insurance partners. They have shown that the efficient network is such that each agent is indirectly connected to each other, leading to the maximal level of insurance in the population, while strategic agents form networks involving income pooling in smaller groups, because the gain for an agent from adding new insurance partners to the group is decreasing with the size of the group while its cost is constant. They have thus provided another theoretical explanation of the observation that full income pooling is not achieved in rural areas of developing countries, but they fail to explain why risk-sharing occurs within networks involving agents having common characteristics.

This paper analyzes which pattern of insurance relationships emerges in the long run when agents are farsighted, rather than myopic, in the sense that they are able to forecast how other agents would react to their choice of partners. In his survey of models of network formation, Jackson (2005) provides support to this behavioral assumption by mentioning that farsightedness is important when agents have good information about each other, which we suspect is the case in the aforementioned communities.

We find that for small costs of establishing and maintaining a partnership, farsighted agents may form efficient networks that involve full income pooling while myopic agents form networks connecting fewer individuals. Two mechanisms explain this result: (i) Farsighted agents belonging to small groups may decide to create new partnerships that are not directly profitable to them, because they realize that other partners will further join this bigger and more attractive group. In other words, the farsightedness of the agents may solve a coordination problem. (ii) Farsighted agents may refrain from deleting costly links if they belong to a big group, as they understand that this may induce others to rearrange their partnerships in a way that deters the myopic incentives to delete the link at first. Farsightedness may thus reconcile the theory with the data observed in small communities. This conclusion does not hold for all cost values. In particular, for very small cost of link formation, myopic agents form efficient networks only while farsighted agents may form inefficient networks. These inefficient networks nonetheless involve full risk-sharing, but not at the minimal cost.

To our knowledge, no existing work has attempted to establish whether agents are farsighted or not when creating their network in rural areas of developing countries. This paper offers a characterization of farsightedly and myopically stable networks that could be used in future work to estimate the degree of farsightedness of agents by comparing the observed networks with the predicted ones under the two different behavioral assumptions.

Appendix A. Proofs.

The following lemma is useful in establishing Proposition 1.

Lemma A.1. (Bramoullé and Kranton, 2007a) For all $g' \in \mathbb{G} \setminus G^*$, we have $g \in M(g')$ for some $g \in G^*$.

Proof. Take a network $g' \in \mathbb{G} \setminus G^*$. Start with g' and let agents successively delete unnecessary links (links connecting agents who are indirectly connected) until a minimally connected network g'' is reached. Then, let some pair of agents belonging to different components of size smaller than s^* add a link between them. Repeating this operation leads to a network g''' in which less than two components have a size smaller than s^*. From g''', take successively a component of size strictly bigger than s^* and let an agent from this component delete a link with an agent who has exactly one link in that component. When all these links are deleted, we end up at network g'''' such that if $(S, h) \in C(g'''')$, then either (i) $\#S = 1$ or (ii) $\#S = s^*$, or (iii) $1 < \#S < s^*$ and no other component $(S', h') \in C(g'''')$ satisfies $1 < \#S' < s^*$. From g'''', build a sequence of networks where at each step k, a link is added between a singleton and an agent belonging to the biggest component of size strictly smaller than s^* in the network of that step g^k. When all these links have been added, we end up in a network $g \in G^*$. Each move in the sequence of networks going from g' to g is profitable, establishing the result. ■

Proof of Proposition 1.

Let G be the pairwise myopically stable set. Suppose that $s^* = s^{**}$ or $s^* > s^{**}$ and $n \leq s^{**}+s^*$, so that G^* consists only of pairwise stable networks. By Lemma A.1, there is a myopic improving path from every network outside G^* to some pairwise stable network in G^*. However, the converse does not hold since each network in G^* is by itself a closed cycle. This establishes that there are no other closed cycle than the pairwise stable networks, that is $G = G^*$. If on the other hand $s^* > s^{**}$ and $n > s^* + s^{**}$, then no pairwise stable network exists. By Lemma A.1, we have that $G \cap G^* \neq \varnothing$. Let $g^* \in G \cap G^*$. Starting from the network g^*, we can reach any network $g \in G^*$ by adding links between members of different components of size s and s' where $s, s' \in \{s^{**}+1, s^*\}$, by letting agents delete links with peripheral agents from components connecting more than s^* agents, and by adding links between a singleton and a member of a component connecting less than s^* agents. Since each of the suggested move is profitable for the agents involved, we have $g \in M(g^*)$ for all $g \in G^*$. We then conclude that $G^* \subseteq G$. ■

The following lemma is used in the proof of Proposition 2.

Lemma A.2. Let a network g be such that $(S,h) \in C(g)$, $\#S > \overline{s}/2$ and $i \in S$. If $U_i(g+ij) > U_i(g)$, then $U_j(g+ij) < U_j(g)$ for all $j \in N$.

Proof. Let $g \in \mathbb{G}$ be such that (S,h),$(S',h') \in C(g)$ and $\#S > \overline{s}/2$. Take agent $i \in S$ and agent $j \in S'$. (i) If $\#S = \#S' > \overline{s}/2$, then we have that $U_i(g+ij) < U_i(g)$ and $U_j(g+ij) < U_j(g)$ by definition of $\overline{s}$. (ii) If $\#S' > \#S \; > \overline{s}/2$, then at least agent $j \in S'$ is not willing to add the link ij. Indeed, by definition of $\overline{s}$, she would not be willing to add a link with an agent of another component of size $\#S'$. She is thus not willing to add a link with agent i who belongs to a component of size smaller than $\#S'$. (iii) Similarly, if $\#S > \#S'$, then at least agent $i \in S$ is not willing to add the link ij.■

Proof of Proposition 2.

Let G be the pairwise myopically stable set.

(i) By contradiction, suppose that a network $g' \in G$ is such that $g' \notin G^m$. Take a network g composed of minimally connected components obtained from g' by deleting unnecessary links. Formally, g is such that $g \in G^m$, and for all $(S',h') \in C(g')$, we have $(S',h) \in C(g)$ for some $h \subseteq h'$. Starting from g' and letting agents delete successively a link that belong to g' but not to g, we find that $g \in M(g')$. Thus the network g belongs to the same closed cycle C as the network g'. However, $g' \notin M(g)$, contradicting the fact that C is a closed cycle. Thus $g' \notin G$.

(ii) By contradiction, suppose that $g' \in G$ and $(S,h) \in C(g')$ with $\#S > \overline{s}$. Since $\#S > \overline{s} \geq s^*$, there is an agent $i \in S$ willing to cut a link ij where $d_j(g') = 1$ (such link exists since $g' \in G^m$ by part (i)). Thus, $g' - ij \in M(g')$, implying that g' and $g' - ij$ are in the same closed cycle C. However $g' \notin M(g' - ij)$ since every path going from $g' - ij$ to g' implies, at some step, that an agent belonging to a component of size bigger than $\overline{s}/2$ should add a link. By Lemma A.2, this move is not profitable. This in turn contradicts the fact that C is a closed cycle. It follows that $g' \notin G$.

(iii.a) By contradiction, suppose that $g' \in G$ such that $d(g') = s^* - 1 - t \geq 0$, where $t \in \mathbb{N}_0^+$. Notice that each network whose total number of links is strictly smaller than $s^* - 1$ is composed of components of size strictly smaller than s^*. By definition of s^*, it follows that every pair of agents can profitably add a link from the network g', i.e. $g' + ij \in M(g')\ \forall ij \notin g'$. Thus g' and $g' + ij$ are in the same closed cycle C. However $g' \notin M(g' + ij)$ since every path going from $g' + ij$ to g' involves, at some step, the deletion of a link kl from a network g'' such that $d(g'') = d(g') + 1$. Since the network g'' is composed of components of size smaller than or equal to s^*, the deletion of the link kl is neither profitable for agent k, nor for agent l. This contradicts the fact that g' and $g' + ij$ are in the same closed cycle. Thus $g' \notin G$.

(iii.b) Take a network $g \in G$. We show that $d(g) \leq n - 1 - \text{int}((n-1)/\overline{s})$. By part (i) of this proposition, we have that $d(g) = n - \#C(g)$. By part (ii) of this proposition, there are no networks in G with fewer components than a network composed of a maximal number of components of size $\overline{s}$ and of one component with the remaining agents. Let g' be such a network. We have $\#C(g') = 1 + \text{int}((n-1)/\overline{s})$, implying that $d(g) \leq d(g') = n - 1 - \text{int}((n-1)/\overline{s})$. ■

The following lemma provides sufficient conditions to have a farsighted improving path from one network to an adjacent one. It implies that the addition of a link from a network such that at least one deviating agent is strictly worse off in the resulting network, or the deletion of a link from a network such that one of the two agents involved in the link is strictly worse off in the resulting network while the other agent is not strictly better off, are deviations that are deterred.

Lemma A.3. Let $g \in \mathbb{G}$. If $U_i(g + ij) < U_i(g)$, then $g \in F(g + ij)$. If $U_i(g - ij) < U_i(g)$ and $U_j(g - ij) \leq U_j(g)$, then $g \in F(g - ij)$.

Proof. Trivial.

Proof of Proposition 3.

Let $0 < c \leq u(n) - u(n-1)$. Notice that for such costs, two agents i, j who are not indirectly connected at a network g are both better off at the network $g + ij$ than at the network g.

(a) Each farsighted improving path emanating from a connected line $g \in G^L$ reaches a connected network, since g Pareto dominates g' for each $g' \in G^m \setminus$

G^M. Thus, a set of networks G' that does not contain a connected network does not satisfy condition (ii) of Definition 2.

(b) Take a network $\widetilde{g} \in G^M$ and let $G = \{g \in \mathbb{G} \mid U_i(g) = U_i(\widetilde{g}) \text{ for all } i \in N\}$. In order to prove that G is a pairwise farsightedly stable set, we will show that (b.i) $\widetilde{g} \in F(g')$ for every $g' \in \mathbb{G} \setminus G$ and (b.ii) $F(g) \cap G = \emptyset$ for every $g \in G$ and hence, Theorem 3 in Herings, Mauleon and Vannetelbosch (2009) applies.

(b.i) Take $g' \in \mathbb{G} \setminus G$. (b.i.1) If for all $g \in G$, we have $g \not\subseteq g'$, then start from g' and build a sequence of networks where at each step, an agent who has more links than at the network $\widetilde{g}$ cuts a link. When all these links have been deleted, we end up at the network g'' such that $d_i(g'') \leq d_i(\widetilde{g})$ for all $i \in N$. Notice that there are at most $n-1$ links in g'' and that $U_i(g'') < U_i(\widetilde{g})$ for some $i \in N$, since $\widetilde{g}$ is efficient and $g'' \notin G$. Agent i cuts one link in g'' leading to the network g''', which is composed of multiple components. In g''' and in the successive networks, an agent who has l links in a component of size s cuts a link, looking forward to the formation of the network $\widetilde{g}$, if she has $l+x$ links or less in $\widetilde{g}$ and $n \geq s+x$. The network reached through this path is $g^{\emptyset}$. Once in $g^{\emptyset}$, agents successively add links to form $\widetilde{g}$. Notice that $u(n) - u(n-1) \geq c$ implies $u(s) - lc \leq u(s+x) - (l+x)c$, if $s + x \leq n$, since the expected utility function is increasing at a decreasing rate. Each agent i cutting a link in a network g in the path where she has l links in a component of size s is willing to do so since her payoff in $\widetilde{g}$ is $U_i(\widetilde{g}) \geq u(n) - (l+x)c > U_i(g)$ Agents adding links from $g^{\emptyset}$ to $\widetilde{g}$, looking forward to the formation of $\widetilde{g}$, are better off in the end network. (b.i.2) If $g \subset g'$ for some $g \in G$, then $d_i(g') \geq d_i(\widetilde{g})$ for all $i \in N$, and $d_j(g\prime) > d_j(\widetilde{g})$ for some $j \in N$. From g', let a pair of agents add a link such that at least one of the two agents adding the link has strictly more links at the current network than at $\widetilde{g}$. By repeating this step, agents reach the complete network g^N. Once there, they successively delete the links that are not in $\widetilde{g}$. Each move of the path is profitable for the deviating agents who are looking forward to the formation of $\widetilde{g}$. We thus conclude that $\widetilde{g} \in F(g')$ for every $g' \in \mathbb{G} \setminus G$.

(b.ii) For every $g \in G$, $F(g) \cap G = \emptyset$ since $U_i(g) = U_i(\widetilde{g})$ for all $g \in G$.

(c) Let $G = \{g_1, g_2\}$, where $g_1, g_2 \in G^M$ such that $U_i(g_1) \neq U_i(g_2)$ for some agent

$i \in N$, and g_1 and g_2 are not star networks. In order to prove that G is a pairwise farsightedly stable set, we will show that it satisfies conditions (i), (ii) and (iii) of Definition 2.

(c.i) Every deviation from a network in the set is deterred by application of Lemma A.3.

(c.ii) We have shown in part (b.i) that $g_1 \in F(g')$ for every $g' \in \mathbb{G} \setminus G^1$, where $G^1 = \{g \in \mathbb{G} \mid U_i(g) = U_i(g_1)$ for all $i \in N\}$. By part (b.i), we have that $g_2 \in F(g')$ for every $g' \in \mathbb{G} \setminus G^2$, where $G^2 = \{g \in \mathbb{G} \mid U_i(g) = U_i(g_2)$ for all $i \in N\}$. Since $G^1 \cap G^2 = \emptyset$, we have $F(g') \cap G \neq \emptyset$ for all $g' \notin G$.

(c.iii) Suppose that some subset $G' \subset G$ is a pairwise farsightedly stable set. Without loss of generality, suppose that $G' = \{g_1\}$. Then, G' does not satisfy the condition (ii) of Definition 2 since $g_1 \notin F(g')$ for $g' \in G^1 \setminus \{g_1\}$, a contradiction. ■

The following two lemmas are central to the proof of Proposition 4.

Lemma A.4. $G^L \subseteq F(g^\emptyset)$ if $c < \min\{u(n) - u(\text{int}((n+1)/2)), (u(n) - u(1))/2\}$.

Proof. For such costs, two agents i and j in different components of size $\text{int}((n+1)/2)$ and $\text{int}(n/2)$ at a network g' both prefer the network $g' + ij$ to the network g'. Take a network $g \in G^L$. Let $g' = g - ij$ where $ij \in g$, $i \in Ce(g)$ and $j \in Ce(g)$ if n is even such that the network g' is composed of one component of size $\text{int}((n+1)/2)$ and of another of size $\text{int}(n/2)$. The following path is a farsighted improving path from $g^\emptyset$ to g. From $g^\emptyset$, add successively each link $kl \in g'$ until the network g' is formed. Agents i and j then add the link ij. Let g'' be a network of the path going from $g^\emptyset$ to g in which agent $k \in N$ adds a link. If $d_k(g'') = 0$, then since $c < (u(n) - u(1))/2$, agent k prefers to add a link, looking forward to the formation of the network g. If $d_k(g'') = 1$, it is profitable for agent k to add a link looking forward to the network g since she belongs to a component of size $s \leq \text{int}((n+1)/2)$ in g'', implying that $U_k(g'') = u(s) - c \leq u(\text{int}((n+1)/2)) - c < u(n) - 2c = U_k(g)$. Since g was chosen arbitrarily, we have that $g \in F(g^\emptyset)$ for all $g \in G^L$. ■

Lemma 3.A.5. For $G = \{g \in G^L \mid U_i(g) = U_i(\widetilde{g})$ for all $i \in N$ for some $\widetilde{g} \in G^L\}$, we have that $G \subseteq F(g')$ for all $g' \notin G$ if $c < \min\{u(n) - u(\text{int}((n+1)/2)), (u(n) - u(1))/2\}$.

Proof. Take a network $\widetilde{g} \in G^L$ and let $G = \{g \in G^L \mid U_i(g) = U_i(\widetilde{g})$ for all $i \in N\}$. Suppose that $c < \min\{u(n) - u(\text{int}((n+1)/2)), (u(n) - u(1))/2\}$. Take $g' \in \mathbb{G} \setminus G$. (i.1) Suppose that $d_i(g') \geq d_i(\widetilde{g})$ for all $i \in N$. Since $g' \notin G$, we have that $d_j(g') > d_j(\widetilde{g})$ for some $j \in N$. From g', let agent j successively add a link with the agents she is not directly connected to. Then, each pair of agents who are not directly connected adds a link between them to form the complete network. From the complete network, the agents cut the links that do not belong to the network $\widetilde{g}$ to reach it. One can see that this sequence of actions describes a farsighted improving path from the network g' leading to the network $\widetilde{g}$. (i.2) Suppose that $d_i(g') < d_i(\widetilde{g})$ for some $i \in N$. Then, start from g' and build a sequence of networks where at each step, an agent who has more links than at the end network $\widetilde{g}$ cuts a link. When all these links have been deleted, we end up at the network g'' such that $d_i(g'') \leq d_i(\widetilde{g})$ for all $i \in N$ and $d_j(g'') < d_j(\widetilde{g})$ for some $j \in N$. It follows that the network g'' is composed of multiple components. In g'' and in the successive networks, the agents having 2 links cut one link. They are willing to do so since they are looking forward to $\widetilde{g}$ where they belong to a bigger component and pay at worse the same cost. When all these links have been deleted, we are in a network composed of components of size 2 and of singletons. Agents having a link successively cut this link to reach $g^{\emptyset}$ looking forward to $\widetilde{g}$. From $g^{\emptyset}$, there is a farsighted improving path leading to $\widetilde{g}$ (Lemma A.4). We conclude that $\widetilde{g} \in F(g')$. This does not depend on the choice of the network $\widetilde{g} \in G$, thus $G \subseteq F(g')$. ■

Proof of Proposition 4.

Suppose that $c < \min\{u(n) - u(\text{int}((n+1)/2)), (u(n) - u(1))/2\}$.

(a) Take a network $\widetilde{g} \in G^L$ and let $G(\widetilde{g}) = \{g \in G^L \mid U_i(g) = U_i(\widetilde{g})$ for all $i \in N\}$. From Lemma A.5, we have that $G(\widetilde{g}) \subseteq F(g')$ for every $g' \in \mathbb{G} \setminus G(\widetilde{g})$. In addition, $F(g) \cap G(\widetilde{g}) = \emptyset$ for every $g \in G(\widetilde{g})$, since $U_i(g) = U_i(g')$ for all $g, g' \in G(\widetilde{g})$. Hence, Theorem 3 in Herings, Mauleon and Vannetelbosch (2009) applies.

(b) Take the set $\{g_1, g_2\}$ where $g_1, g_2 \in G^L$ and $U_i(g_1) \neq U_i(g_2)$ for some $i \in N$. To show that $\{g_1, g_2\}$ is a pairwise farsightedly stable set, we show that it satisfies the three conditions of definition 2. (ii.1) Deterrence of external deviations is satisfied since the network g' reached by adding or deleting a link from g_1 (or g_2) is not a minimally connected line. Thus, from Lemma A.5, $g_1 \in F(g')$ which deters the incentives to deviate from g_1. (ii.2) External stability is ensured by Lemma A.5 since $g_1 \in F(g')$ for the networks g' such that $U_i(g') \neq U_i(g_1)$ for some agent i, and $g_2 \in F(g')$ for the networks g' such that $U_i(g') = U_i(g_1)$ for all $i \in N$. (i.3) Minimality is ensured since external stability would be violated if the set was smaller. ■

The following lemma is used in the proof of Proposition 5.

Lemma A.6. If n is odd, then for all $g' \in \mathbb{G} \backslash G_k$, where $G_k = \{g \in \mathbb{G} \mid d_i(g) = 1$ for all $i \in N \setminus \{k\}$, $d_k(g) = 0\}$, we have that $G_k \subseteq F(g')$ if $\max\{(u(3) - u(1))/2, u(n) - u(2)\} < c < u(2) - u(1)$.

Proof. Let n be odd and let $\max\{(u(3) - u(1))/2, u(n) - u(2)\} < c < u(2) - u(1)$. Take $k \in N$ and let $G_k = \{g \in \mathbb{G} \mid d_i(g) = 1$ for all $i \in N \setminus \{k\}$, $d_k(g) = 0\}$. Take $g \in G_k$ and $g' \in \mathbb{G} \setminus G_k$. Notice that an agent prefers a network in which she has one link to any network in which she has two or more links when $u(n) - u(2) < c < u(2) - u(1)$.

(i) If $d_i(g') \leq 1 \; \forall i \in N$, then start with g' and build a sequence of networks where at each step, either a singleton adds a link that belongs to the network g, or an agent who has two links deletes a link that does not belong to the network g until the network g is reached. Step $1a$: A singleton in g' other than agent k adds a link that belongs to the network g. Since at least one agent, say i, has no link at g' then $U_i(g') = u(1) < U_i(g) = u(2) - c$, thus agent i is willing to add the link looking forward to g. The other agent, say j, has either no link at g' or she has one link in a component of size 2. In both cases, she agrees to add the link ij looking forward to g. Step $1b$: In the remaining network, if an agent has two links, she deletes a link that does not belong to g. This agent is willing to delete a link looking forward to g where she has one link. Step l: Proceed inductively in l, if an agent other than agent k is a singleton, she adds a link that belongs to g; then, on the remaining network, if an agent has two links, she deletes a link that does not belong to g. Step L: When all these links are added or removed, we end up at the network g. We conclude that $g \in F(g')$. Since the choice of $g \in G_k$ does not matter, we conclude that $G_k \subseteq F(g')$.

(ii) If $d_i(g') > 1$ for some agent $i \in N$, then start with g' and build a sequence of networks where at each step, some agent other than agent k who has more than one link deletes a link. When all these links have been deleted, if agent k has more than one link, she successively deletes all her links but one so that the network g''' is reached with $d_i(g''') \leq 1 \ \forall i \in N$. From g''', there is a farsighted improving path going to g (see part (i) of the proof of this lemma). Step $1a$: An agent, say i, who has more than one link deletes a link ii' such that $i' \neq k$ and agent i' has exactly one link at the current network. Repeat this step until a network is reached in which the agents having more than one link are connected to agents having also more than one link or to agent k. Step $1b$: In the remaining network, let an agent, say j, who has more than one link, delete a link different than the link jk. An agent deleting a link in one of those steps is willing to do so as she has at least 2 links at the network where she deletes a link and she is looking forward to g in which she has one link. Step l: Proceed inductively in l, each time an agent, say i, with two links or more is connected to an agent other than agent k that has exactly one link, agent i deletes that link. Then, on the remaining network, an agent who has two links or more, say agent j, deletes a link other than the link jk. Step L: When all these links have been deleted, a network g'' is reached such that $d_i(g'') \leq 1$, $\forall i \in N \setminus \{k\}$. Step $L+1$: If agent k has 2 links or more, she successively deletes all her links but one. Agent k has s links in a component of size $s+1$ for some $s \geq 2$ at a network, say g_1, where she deletes a link. She is willing to delete a link in g_1 looking forward to g since, for $s \geq 2$, we have that $U_k(g_1) = u(s+1) - sc \leq u(3) - 2c < U_k(g) = u(1)$ when $u(3) - u(1) < 2c$. Step $L+2$: We are in a network g''' such that $d_i(g''') \leq 1$ for all $i \in N$ and $g''' \notin G_k$ by construction. In part (i) of this proposition we have shown that $G_k \in F(g''')$. We conclude that $G_k \in F(g')$. ■

Proof of Proposition 5.

Let $u(n) - u(2) < c < u(2) - u(1)$.

(a) Suppose that n is even. Let $G = \{g \in \mathbb{G} \mid d_i(g) = 1 \text{ for all } i \in N\}$. In order to prove that G is the unique pairwise farsightedly stable set, we will show that (a.i) for every $g' \in \mathbb{G} \setminus G$, we have that $F(g') \cap G \neq \emptyset$ and (a.ii) for every $g \in G$, $F(g) = \emptyset$ and hence, Theorem 5 in Herings, Mauleon and Vannetelbosch (2009) applies.

(a.i) Take $g' \in \mathbb{G} \setminus G$. Start with g' and build a sequence of networks where at each step, either an agent with more than one link deletes a link, or two unconnected agents add a link between them. This path leads to the formation of a network $g'' \in G$ and each deviating agent is better off in g'' than in the networks in which she adds/cuts a link. Thus $F(g') \cap G \neq \emptyset$.

(a.ii) By contradiction, suppose that $F(g) \neq \emptyset$, say $g' \in F(g)$, for some network $g \in G$. Then, at least an agent, say i, is willing to create or delete a link from g looking forward to g', that is, $U_i(g) < U_i(g')$. Having $U_i(g) < U_i(g')$ implies that agent i has exactly one link in g' and belongs to a component of size strictly bigger than 2. Then, at least an agent, say j, has 2 links or more in g'. However, every path going from g to g' is such that the payoff of agent j is smaller in g' than in the network in which she adds a second link. This contradicts the fact that $g' \in F(g)$. Thus $F(g) = \emptyset$.

(b) Suppose that n is odd and that $(u(3) - u(1))/2 < c$. We have that (b.i) for every $g' \in \mathbb{G} \setminus G_k$, $F(g') \cap G_k \neq \emptyset$ (see Lemma A.6), and (b.ii) for every $g \in G_k$, $F(g) \cap G_k = \emptyset$, since $U_i(g) = U_i(g')$ for all $i \in N$, for all $g, g' \in G$. Hence, Theorem 3 in Herings, Mauleon and Vannetelbosch (2009) applies, G_k is a pairwise farsightedly stable set.

(c) Suppose that n is odd, $n \geq 5$ and $(u(3)-u(1))/2 < c$. Take $g_k \in G_k$ and $g_l \in G_l$ such that $k \neq l$. In order to prove that $G = \{g_k, g_l\}$ is a pairwise farsightedly stable set, we show that G satisfies the three conditions of Definition 2.

(c.i) Every deviation from a network in the set is deterred by application of Lemma A.3.

(c.ii) We have shown in Lemma A.6 that $g_k \in F(g')$ for every $g' \in \mathbb{G} \setminus G_k$ and $g_l \in F(g')$ for every $g' \in \mathbb{G} \setminus G_l$. Since $G_l \cap G_k = \emptyset$, we thus have $F(g') \cap G \neq \emptyset$ for all $g' \notin G$.

(c.iii) Suppose by contradiction that some subset $G' \subset G$ is a pairwise farsightedly stable set. Without loss of generality, suppose that $G' = \{g_k\}$. We then have that condition (ii) of Definition 2 is not satisfied since $g_k \notin F(g')$ for

$g' \in G^k \setminus \{g_k\}$, and $G^k \setminus \{g_k\} \neq \{\emptyset\}$ when $n \geq 5$, contradicting the fact that G' is a pairwise farsightedly stable set. Thus, G satisfies the minimality condition. ■

Appendix 3.B. Description of the algorithm

Having associated to each network a number between 1 and K, the output of the algorithm is (i) a square matrix F of dimension $K \times K$, where K is the total number of networks among n agents, such that $F(i,j) = 1$ if there is a farsighted improving path from network number $i \in \{1, K\}$ leading to network $j \in \{1, K\}$ and (ii) a matrix $PFFS$ of dimension $L \times K$, where L is the total number of pairwise farsightedly stable sets of networks such that a set composed of networks associated with non-zero elements of a line is a pairwise farsightedly stable set of networks.

To determine whether there is some farsighted improving path from some initial network $a \in \{1, K\}$ to some final one $b \in \{1, K\}$, the algorithm proceeds by steps.

Step 1: The algorithm creates at the first step a vector $G_{ab}(1)$ containing all the possible networks that are adjacent to the initial network a such that either one link is added and both agents are better off in the final network b compared to the initial one, or one link is deleted and at least one agent involved in that link is better off at the end network.

Step 2: At the second step, the algorithm creates a vector $G_{ab}(2)$ which consists of the elements of $G_{ab}(1)$ and all the networks that are adjacent to a network c in $G_{ab}(1)$ but not yet included in $G_{ab}(1)$ and such that either one link is added and both agents are better off in the final network b compared to the current network c, or one link is deleted and at least one agent involved in that link is better off at the end network.

Step p: At the p^{th} step, the algorithm creates a vector $G_{ab}(p)$ which consists of the elements of $G_{ab}(p-1)$ and all the networks that are adjacent to a network c in $G_{ab}(p-1)$ but not yet included in $G_{ab}(p-1)$ and such that either one link is added and both agents are better off in the final network b compared to the current network c, or one link is deleted and at least one agent involved in that link is better off at the end network.

The algorithm stops at step P where P is the smallest integer such that either $b \in G_{ab}(P)$, or $G_{ab}(P) = G_{ab}(P-1)$. The finiteness of the number of networks implies that the algorithm ends in a finite number of steps. There is a farsighted improving path from the initial network to the final network if the final network b belongs to $G_{ab}(P)$. By running this algorithm for each possible pair of initial network and final network, we obtain a square matrix F of dimension $K \times K$, where $K = \Sigma_{i=0}^{n(n-1)/2} C_i^{n(n-1)/2}$ is the total number of networks among n agents, such that $F(a, b) = 1$ if there is a farsighted improving path from the network a leading to the network b, and $F(a, b) = 0$ otherwise.

In the second part of the algorithm, we build a matrix $PFSS$ of dimension $L \times K$, where L is the total number of pairwise farsightedly stable sets of networks such that a set composed of networks associated with non-zero elements of a line is a pairwise farsightedly stable set of networks. To find this matrix, we start from a matrix H which contains all the possible equilibrium candidates -each line of the matrix being associated with one candidate- and we successively delete the lines that do not satisfy external stability, deterrence of external deviations, and minimality.

Step 1: First build a matrix H of size $\Sigma_{i=1}^{K} C_i^K \times K$, such that each line of H corresponds to a different equilibrium candidate. When $n = 3$, we have 8 different networks that can form 256 different equilibrium set candidates. A number between 1 and 8 is attributed to each network. For candidates of less than 8 networks, we use 0 to fill in the matrix. For example, the first line of H is composed of the number 1 in the first cell and of zeros in the remaining ones, and corresponds to the candidate where the empty network (who is associated with the number 1) as a singleton is a candidate. For the sake of notation, let $l_M(k)$ be the set of the non-zero elements of line k of the matrix M. We thus have $l_H(1) = \{1\}$.

Step 2: Build the matrix H' obtained from H by deleting the lines that do not satisfy the external stability requirement. To do so, look for each line k of the matrix H whether $F(a, b) = 1$ for all $a \in \{1, K\} \setminus l_H(k)$, for some $b \in l_H(k)$.

Step 3: Build the matrix H'' obtained from H' by deleting the lines that do not satisfy deterrence of external deviations. To do so, look for each line k of the matrix H' whether for all network $a \notin l_{H'}(k)$ obtained from a network $b \in l_{H'}(k)$ by adding a link ij, we have $F(a, c) = 1$ for some $c \in l_{H'}(k)$ such that either $Y_i(c) < Y_i(b)$, or $Y_j(c) < Y_j(b)$, or $Y_i(c) = Y_i(b)$ and $Y_j(c) = Y_j(b)$. Then repeat the operation for deviations involving the deletion of a link.

Step 4: Build the matrix $PFSS$ from H'' by removing the lines that do not satisfy the minimality requirement.

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