The Timing of Contests

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Abstract

We develop a two-period model of contest with endogenous military investment decisions and shifting contest-effort productivities. We demonstrate that sufficient conditions for having delay in win contests is that players at least partially discount the future or that the costs associated with the contest be lower in the future for both players. If the relative contest productivity of the more efficient contestant increases through time, short run settlements can be reached at equilibrium. We also show that, contrary to the previous findings, settlements can become more likely for higher discount rates, provided the contestants' relative productivity of contest effort evolves over time.

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1 Introduction

When property rights over a resource fail to be perfectly defined and enforced, agents devote resources to claim ownership over the prize and confront each other in a contest. Such situations may be encountered in political contests, in rent seeking activities, in sports contests, in R&D competition, in military confrontations, or in legal disputes, to name but some. Many properties of contests have extensively been explored by scholars, including the general properties of contests technologies (Skaperdas 1996, Hwang 2011), of contest designs (Moldovanu and Sela 2001, 2006), or of the link between inequality concepts and contests (Hodler 2006, Esteban and Ray 2011). The focus of our paper on the dynamic properties of contests.

The static properties of contests have been widely explored over the past twenty years. Despite some early work by Garfinkel (1990), it is essentially more recently that scholars have systematically explored the dynamic features of contests (see Konrad 2009 for an overview). Garfinkel (1990) demonstrates how settlements may be sustained with trigger strategies where deviators (i.e. contest initiators) are punished by reversion to permanent contest equilibria. Even in the absence of such behavioral assumptions, however, Konrad (2012) points at the "discouragement effect" that could help sustain peace. In dynamic settings the expected future interaction between contestants is likely to reduce the present discounted benefits from fighting, therefore incentivizing players to contain or even totally forgo investments in the contest.

Some contributions have combined the dynamic feature with an endogenous shifting power (Skaperdas 1996, Garfinkel and Skaperdas 2000, Powell 2012). Both Skaperdas (1996) and Garfinkel and Skaperdas (2000) show that when present victories translate into a future strategic advantage players are further induced to initiate conflict in the short run. Powell (2012) takes the players conflict investments as given and integrally focuses on the power-shifting aspect of conflict in an infinite horizon game. His main contribution is the possibility of persistent conflict because of the players' willingness to retain their strategic advantage in the future.

The literature reviewed above relies on players continuing to have strategic interactions in the future irrespectively of the contest outcome. While this assumption is well suited for a wide range of contests, some confrontations result in a contestant being excluded from future interactions. Reuveny et al. (2011) underline the important difference between what they term "win" (decisive) and "share" (non-decisive) contests in dynamic settings. Indeed, while the expected utility under both types of confrontations is the same for risk-neutral agents in static settings, the dynamic incentives can significantly differ across scenarii since continuation payoffs differ. A series of papers has shown that in settings with "win" contests the incentives to initiate contests are higher in dynamic games compared to the same game with a full discount, i.e. compared to static games (Skaperdas 2006, McBride and Skaperdas 2007). In dynamic "win" contests, victory maps into a longer period of time over which to enjoy the pie at stake. Since this translates into a higher marginal benefit of effort in the confrontation, players invest more resources in the contest. From this stems the conclusion that higher discount rates - which increase the dynamic dimension of the game - are associated with more rent-dissipation in contests, and lower likelihoods of reaching settlements. Considering situations in which the likelihood of winning the contest evolves over time, Bester and Konrad (2004) show that dynamic nature of the game could contribute to achieving settlement in the short run: a contest may be deferred to the second period provided attacking is less efficient than defending.

Bester and Konrad (2004) point at the important phenomenon of opposing parties in armed confrontations postponing the contest. A limitation of their argument is that it relies integrally on the asymmetry between attack and defense. In contexts of armed confrontation, this argument seems at first uncontested since as Hirshleifer (2000) reminds us, the 3 : 1 ratio is a 'familiar military rule of thumb'; the offender needs as much as three times the strength of the defender to defeat its foe. Hirshleifer, however, also underlines that while this rule of thumb is fairly verified at the time of the military clash, the offender also has strategic advantages to the extent that he can carefully choose the place and time of the confrontation, an argument put forward by Bismark who declared that 'No government, if it regards war as inevitable even if it does not want it, would be so foolish as to leave to the enemy the choice of time and occasion and to wait for the moment which is the most convenient for the enemy' (Levy: 99, 1997). Levy (1984), Biddle (2001), or more recently Gortzak et al. (2005) vividly advise scholars to be extremely cautious when constructing the entire explanatory mechanism of war on the offense-defense balance.

The literature on "win" contests highlights two opposing effects of dynamics on the short run outcome of a game with contest. In this paper we propose a comprehensive and unifying framework for two period games featuring two players, that enables us to identify the multiple factors conducive to delay (Bester and Konrad) or rushing in contests (Skaperdas 2006, McBride and Skaperdas 2007). As a general rule, we demonstrate that sufficient conditions for having delay in win contests is that players at least partially discount the future or that the costs associated with the contest be lower in the future for both players. When the players effort choice in the contest is modeled as an endogenous variable, we show that if the relative contest productivity of the more efficient contestant increases through time, the players' aggregate effort is higher in the short run, thus creating scope for short run settlements. We therefore complement the work of Bester and Konrad (2004) by replicating their main results (existence of delay in contests) without exogenously granting a strategic advantage to defense. We also demonstrate that, contrary to the findings of Skaperdas (2006) and McBride and Skaperdas (2007), settlements can become more likely for higher discount rates, provided the contestants' relative productivity of contest effort evolves over time.

The rest of the paper is organized as follows. In the next section we introduce the framework of the model. Sections 3 and 4 respectively solve the game for exogenous and endogenous efforts in contests. In section 5 we propose a discussion of the model and its results, and the last section concludes.

2 The framework

A prize worth W is allocated among two players, i and j. We denote by α_k the fraction of the total prize W allocated to player k. The two players live two periods, and must decide in each period whether they accept the status quo allocation of the prize, or whether they initiate a contest to attempt modifying it. If both players accept the status quo in a period, each player keeps his allocation in that period, and the game moves on to the next one. Otherwise, a contest occurs, and the player who wins it obtains the entire prize for the remaining periods. The players discount their second period payoff according to their discount factor δ_k, they have full information about the game, and are risk-neutral. In addition, the allocations are assumed to be indivisible, so that a player cannot transfer part of his allocation to the other player to avoid a contest.

3 The exogenous model

We develop in this section a model where the probability with which player k wins the prize if a contest occurs in period t is exogenously given, and denote it by $p_{k,t}$. If a contest occurs in period t, either player i or player j wins it, that is $p_{i,t} + p_{j,t} = 1$. In addition, to capture the inefficiencies that typically arise in contests, in this exogenous effort version of the game we assume that the contest involves an exogenous cost $F_{k,t}$ to player k.[1] In each period, both players simultaneously choose whether they accept the status quo or if they contest it. Formally, the strategy of player k in period t is $\sigma_{k,t} \in \{y, n\}$, where y stands for "yes, I accept the status quo allocation", while n stands for "no, I do not accept the status quo allocation". We let $\sigma_k = (\sigma_{k,1}, \sigma_{k,2})$ be the strategy of player k, and σ be the strategy profile (σ_i, σ_j). Abusing notation, we also refer to the strategy profile of the players in period t as $\sigma_t = (\sigma_{i,t}, \sigma_{j,t})$. If a player does not accept the status quo allocation in a period, a contest occurs and each player obtains the prize for the remaining periods with some probability. Otherwise, If no player has contested the status quo in a previous period, and both players accept the status quo in the current period, each player gets his allocation. The utility of player k is given as follows.

$$\begin{cases} U_k(\sigma) = \alpha_k W(1+\delta_k), \text{ if } \sigma_1 = \sigma_2 = (y, y) & (A) \\ \\ U_k(\sigma) = \alpha_k W + \delta_k(p_{k,2}W - F_{k,2}), \text{ if } \sigma_1 = (y, y), \text{ and } \sigma_{l,2} = n \text{ for some } l = i, j & (B) \\ \\ U_k(\sigma) = (1+\delta_k)p_{k,1}W - F_{k,1}, \text{ if } \sigma_{l,1} = n \text{ for some } l = i, j & (C) \end{cases}$$

The game is represented in Figure 1. By backward induction, we characterize the subgame perfect equilibrium (σ^*) of the model.

[1] Contests involve rent-dissipation since otherwise productive resources are devoted to improving one's winning odds (Tullock 1980). Moreover the value of the prize can be reduced because of the destructiveness of a contest (Grossman and Kim 1995, De Luca and Sekeris 2012), or as a consequence of the players' risk aversion (Skaperdas 1991, and Skaperdas and Gan 1995).

Proposition 1. *The subgame perfect equilibria σ^* of the model are given by:*

(i) $\sigma^*_{k,2} = n$ *if* $p_{k,2}W - F_{k,2} > \alpha_k W$; *otherwise,* $\sigma^*_{k,2} = y$.

(ii) Suppose $p_{l,2}W - F_{l,2} \leq \alpha_l W$, l=$\{i,j\}$. *Then* $\sigma^*_{k,1} = n$ *if* $(1+\delta_k)p_{k,1}W - F_{k,1} > \alpha_k W(1+\delta_k)$; *otherwise,* $\sigma^*_{k,1} = y$.

(iii) Suppose $p_{l,2}W - F_{l,2} > \alpha_l W$ *for some* l. *Then* $\sigma^*_{k,1} = n$ *if* $(1+\delta_k)p_{k,1}W - F_{k,1} > \alpha_k W + \delta_k(p_{k,2}W - F_{k,2})$; *otherwise,* $\sigma^*_{k,1} = y$.

These results are obtained simply by comparing the utility of the players under the different scenarii, keeping in mind that the players' decision in period 1 depends on the expected outcome in period 2. If a settlement is expected to occur in period 2, the two players agree to settle in period 1 if both are better off by enjoying their allocation during the two periods than under a contest in period 1. On the other hand, if a contest is expected to occur in period 2, a player agrees on a status quo allocation in period 1 if he is better-off by delaying the contest than by precipitating it.

We focus our attention on case (*iii*) of Proposition 1, and more precisely on the possibility of both contestants playing y in period 1. The theory on dynamic contests has shown that if a party's expected benefit increases in the future, it is necessary that the opponent's expected benefit shrinks, eventually increasing the latter's incentives to preventively initiate a contest (Skaperdas and Syropoulos 1996, Skaperdas 2006, McBride and Skaperdas 2007). The following quote of the military strategist von Clausewitz is very illuminating in that respect:

> If it is in A's interest not to attack B now but to attack him in 4 weeks, then it is in B's interest not to be attacked in 4 weeks' time, but now. (von Clausewitz, 1976 p.84)

Some authors have nevertheless shown that under some circumstances settlements may be reached because of the existence of dynamics (Garfinkel 1990, Bester and Konrad 2004, Jackson and Morelli 2009). The present simple two-period framework enables us to identify the conditions making both players willing to reach a temporary settlement. A necessary condition for a contest to be delayed in period 1 is that the sum of utilities in a delayed contest be higher than in an immediate contest. Formally, the following condition should be satisfied :

$$W + W(\delta_i p_{i,2} + \delta_j p_{j,2}) - \delta_i F_{i,2} - \delta_j F_{j,2} > W((1+\delta_i)p_{i,1} + (1+\delta_j)p_{j,1}) - F_{i,1} - F_{j,1} \quad (1)$$

If Condition 1 does not hold, the aggregate utility under an immediate contest is higher than under a delayed contest, implying that an immediate contest is inescapable. On the other hand, if Condition 1 holds, there exists an allocation of the prize (α_i, α_j) and a vector of success probabilities $(p_{i,1}, p_{j,1}, p_{i,2}, p_{j,2})$ such that a contest occurs in period 2 at equilibrium, since these parameters determine the partition of the prize under a settlement and a contest respectively.

Consider as a benchmark a situation where the players do not discount the future ($\delta_i = \delta_j = 1$), and the cost of a contest is similar for both players and constant over time ($F_{k,1} = F_{k,2} = F$, $k \in \{i, j\}$). In this case, the left-hand side and the right-hand side of equation 1 are equal, so that there is no scope for delay in contest. This result confirms the earlier quote of Von Clausewitz'. Indeed, if player i prefers entering a contest tomorrow rather than avoiding it or fighting today, then player j prefers to wage contest today because players have opposing interests and contest is unavoidable since player i will refuse the settlement in period 2.

The following proposition reveals that to have delay in contest at equilibrium for some parameters configurations, it is sufficient to modify the benchmark scenario by introducing either discounting, or a cost of contest that is decreasing over time,

Proposition 2. *There are subgame perfect equilibria of the model such that* $\sigma_1^* = (y, y)$, $\sigma_2^* \neq (y, y)$ *if (i)* $\delta_i = \delta_j < 1$ *while* $F_{k,1} = F_{k,2} = F$, $k \in \{i, j\}$, *or (ii)* $\delta_i = \delta_j = 1$, *and* $F_{k,1} > F_{k,2}$ *and* $F_l^1 \geq F_l^2$ *for* $k, l \in \{i, j\}, k \neq l$.

This proposition is easily proven since condition 1 is satisfied in the two scenarii mentioned. Notice also that when $\delta_i < \delta_j \leq 1$, delay is possible but only if additional conditions on the evolution of the likelihood of success in a contest hold. Indeed equation 1 can then be rewritten as $(p_{i,2} - p_{i,1}) < (F_i(1-\delta_i) + F_j(1-\delta_j))/R(\delta_j - \delta_i)$. The right hand side is positive, and thus delay is possible if the likelihood that the player having the smallest discount factor (player i in this case) wins the contest does not increase too much from period 1 to period 2.

In light of condition 1, we re-visit the two-period contest model of Bester and Konrad (2004)

Bester and Konrad (2004) develop a model of contest between 2 players, where the likelihood of a success in a contest is higher when the contest is initiated by the other player.[2]Letting the probability of success depend on the identity of the player initiating the contest, we note by $p_{k,t}(l)$ the probability that k wins the contest in period t if l initiates it. Then, a defensive advantage is assumed if $p_{k,t}(l) > p_{k,t}(k)$ where $k, l \in \{i, j\}, l \neq k$, and it implies that the sum of probabilities of success from initiators of a conflict is smaller than 1 $(p_{i,t}(i) + p_{j,t}(j) < 1)$. All else equal, Condition 1 then becomes:

$$2[p_{i,1}(i) + p_{j,1}(j)] - [p_{i,2}(i) + p_{j,2}(j)] < 1$$

This inequality is always satisfied if the sum of probabilities $(p_{i,t}(i) + p_{j,t}(j))$ remains constant over time. In that case, delay is an equilibrium outcome for appropriate parameters of the model. In Bester and Konrad (2004), however, the sum of probabilities is assumed to vary through time. More specifically, they assume that $p_{k,t}(k) = e_{k,t}/(e_{k,t} + de_{l,t})$, for $k, l \in \{i, j\}$, $l \neq k$, where $e_{k,t}$ denotes k's equipment at time t, and $d > 1$ is the parameter that accounts for the defensive advantage. Thus, the sum of probabilities is $p_{i,t}(i) + p_{j,t}(j) = (d(e_{i,t}^2 + e_{j,t}^2) + 2e_{i,t}e_{j,t})/(d(e_{i,t}^2 + e_{j,t}^2) + (1 + d^2)e_{i,t}e_{j,t})$.

[2]Their focus is on armed conflicts. They argue that there are cases where the armies are more efficient in defense.

Notice first that when the ratio of equipment remains constant over time ($e_{k,1} = \beta e_{k,2}$, for $k = i, j$ and $\beta \in \mathcal{R}^+$), the sum of probabilities remains also constant over time and delay is possible. In general, the sum of probabilities is smaller the more equal the equipment of the players, and thus delay is possible as long as the equipment of the weaker does not converge too much to that of the strongest in relative terms.[3] Denoting $(p_{i,1}(i) + p_{j,1}(j)) - (p_{i,2}(i) + p_{j,2}(j))$ by Δp, we have $\Delta p > 0$ if Max$\{\frac{e_{i,2}}{e_{j,2}}, \frac{e_{j,2}}{e_{i,2}}\}$ <Max$\{\frac{e_{i,1}}{e_{j,1}}, \frac{e_{j,1}}{e_{i,1}}\}$. Rewriting equation 1 as $\Delta p < 1 - (p_{i,1}(i) + p_{j,1}(j))$, we see that $\Delta p > 0$ is necessary for delay to be impossible, but not sufficient.

4 The endogenous model

We develop in this section a model where the likelihood of a success in a contest is determined by the effort exerted by the players in order to win the prize. In a given period t, each player k announces in a first stage whether he initiates a contest to attempt appropriating the entire prize. The players' decisions are simultaneous. If a player does not accept the status quo allocation in a period, a contest occurs and each player obtains the prize for the remaining periods with some probability, determined by the effort the players exert in a second stage. Otherwise, If no player has contested the status quo in a previous period, and both players accept the status quo in the current period, each player gets his allocation. As a tie-breaking rule, we assume that in case of indifference the players prefer accepting the settlement. The contest technology is given by a standard contest success function so that if the players respectively exert efforts $e_{k,t}$ and $e_{-k,t}$, the probability that player k eventually obtains the total control of the prize if a contest is initiated in the first stage

[3] Dropping the t subscripts and differentiating the sum of probabilities with respect to e_i yields an expression whose sign is given by $(2e_i d + 2e_j)(d^2-1)e_i e_j - (d^2-1)e_j \left(d(e_i^2 + e_j^2) + 2e_i e_j\right)$, which can be simplified to $(d^2-1)e_j d(e_i^2 - e_j^2)$. Hence, if $e_i > e_j$, increases in e_i increase the sum of probabilities, and vice versa for $e_i < e_j$.

of period t is given by $p_k(e_{k,t}, e_{-k,t}) = \frac{\theta_{k,t}e_{k,t}}{\theta_{k,t}e_{k,t}+\theta_{-k,t}e_{-k,t}}$, where $\theta_{k,t}$ stands for the productivity of player k's effort in a contest in period t. The evolution of the productivity parameter from period 1 to period 2 is exogenously given. In particular, experience in exerting effort in previous periods does not modify the future productivity of effort. We assume that the players are able to enforce a settlement when it is the outcome of the first stage. This assumption may be interpreted as the players having a very strong commitment capacity, or that the deviator incurs important losses in terms of reputation. Also, the effort of the players in a contest only affects the current period allocation of the prize. Finally, the likelihood of success in a contest is solely determined by the effort exerted in the current period. In particular, no accumulation of effort is possible through time.

The strategy of each player consists, in each period, of a choice to accept or contest the status quo in a first stage, and of a level of effort in the second stage, for each action profile chosen in the first stage. Formally, we let S be the state in the second stage of period t when $\sigma_t = (y, y)$, while C is the state in the second stage when $\sigma_{t,l} = n$ for some $l = i, j$. The strategy of player k in period t is $\gamma_{k,t} = (\sigma_{k,t}, e_{k,t}(S), e_{k,t}(C))$, where $\sigma_{k,t} \in \{y, n\}$, and $e_{k,t}(S), e_{k,t}(C) \in \mathcal{R}^+$. We let γ be the strategy profile (γ_i, γ_j).

Assuming that the players have the same discount factor δ, and that the only cost of the contest lies in the rent dissipation due to the players' effort investments, the utility of player k is given as follows.

$$U_k(\gamma) = (1+\delta)\alpha_k W - e_{k,1}(S) - \delta e_{k,2}(S), \text{ if } \sigma_1 = \sigma_2 = (y,y) \tag{2}$$

$$U_k(\gamma) = \alpha_k W - e_{k,1}(S) + \delta(\frac{\theta_{k,2}e_{k,2}(C)}{\theta_{k,2}e_{k,2}(C) + \theta_{-k,2}e_{-k,2}(C)}W - e_{k,2}(C)) \tag{3}$$

$$\text{if } \sigma_1 = (y,y), \text{ and } \sigma_{l,2} = n \text{ for some } l = i,j$$

$$U_k(\gamma) = (1+\delta)\frac{\theta_{k,1}e_{k,1}(C)}{\theta_{k,1}e_{k,1}(C) + \theta_{-k,1}e_{-k,1}(C)}W - e_{k,1}(C) \text{ if } \sigma_{l,1} = n \text{ for some } l = i,j \tag{4}$$

We analyze the subgame perfect equilibrium of the game, proceeding by backwards induction. We write $e^*_{k,t}(S)$ and $e^*_{k,t}(C)$ the equilibrium levels of effort, given the outcome of the first stage in period t. When both players settle in the first stage of a given period, they have no incentives to invest in appropriative activities in the second stage of that period in our framework. We thus have $e^*_{k,t}(S) = 0$ for $k = i,j$ and $t = 1,2$. If a contest is triggered in period 2, both players choose their effort level $e_{k,2}$ in order to maximize 3. Combining the players' reaction functions, the equilibrium amount of effort of both players is:

$$e^*_{i,2}(C) = e^*_{j,2}(C) = e^*_2(C) = \frac{\theta_{i,2}\theta_{j,2}}{(\theta_{i,2} + \theta_{j,2})^2}W \tag{5}$$

If a contest occurs in period 1, player k chooses the level of effort $e_{k,1}$ that maximizes equation (4). Combining the players' reaction functions, the equilibrium amount of effort of both players is:

$$e^*_{i,1} = e^*_{j,1} = e^*_1 = (1+\delta)\frac{\theta_{i,1}\theta_{j,1}}{(\theta_{i,1} + \theta_{j,1})^2}W \tag{6}$$

Notice that player k wins the contest in period t when both players exert the equilibrium choices of effort with probability $p^*_{k,t} = \theta_{k,t}/(\theta_{k,t} + \theta_{-k,t})$.

Consider the strategy profiles $\sigma_{i,1} = \sigma_{j,1} = y$, $\sigma_{l,2} = n$ for some $l = i,j$, $e_{i,1} = e_{j,1} = e^*_1(S)$ and $e_{i,2} = e_{j,2} = e^*_2(C)$, and denoting these profiles by $\gamma^{S,C}$, the payoff of player k under settlement in period 1 and contest in period 2, given equilibrium efforts in both periods is

$$U_k(\gamma^{S,C}) = \alpha_k W + \delta \frac{\theta_{k,2}^2}{\left(\theta_{k,2} + \theta_{-k,2}\right)^2} W \tag{7}$$

Using the same notation, we write $\gamma^{S,S}$ for the strategy profile where both choose to settle in each period and therefore invest no effort at equilibrium, and $\gamma^{C,\cdot}$ for the strategy profiles where one player initiates a contest in period 1 and both choose the equilibrium level of effort $e_{i,1} = e_{j,1} = e_1^*(C)$. We thus have $U_k(\gamma^{S,S}) = (1+\delta)\alpha_k W$, and $U_k(\gamma^{C,\cdot}) = (1+\delta)W\theta_{k,1}^2 / \left(\theta_{k,1} + \theta_{-k,1}\right)^2$. Comparing $U_k(\gamma^{S,C})$ with $U_k(\gamma^{S,S})$, settlement prevails in period 2 at equilibrium if:

$$\alpha_k \geq \frac{\theta_{k,2}^2}{\left(\theta_{k,2} + \theta_{-k,2}\right)^2} \qquad k = i, j$$

The players' decision in the first stage of period 1 depends on the expected outcome in period 2. If a settlement is expected to occur in period 2, the two players agree to settle in period 1 if both are better-off under a durable settlement than under an immediate contest, i.e. if $U_k(\gamma^{S,S}) \geq U_k(\gamma^{C,\cdot})$ for $k = i, j$. On the other hand, if a contest is expected to occur in period 2, a player proposes to settle in period 1 if he is better off under a delayed contest than under an immediate one, i.e if $U_k(\gamma^{S,C}) > U_k(\gamma^{C,\cdot})$.

The next proposition characterizes the subgame perfect equilibria of the game.

Proposition 3. *The subgame perfect equilibria of the model with endogenous effort are characterized by the following expressions*

(i.a) $e_{i,t}^*(S) = e_{j,t}^*(S) = 0$

(i.b) $e_{i,1}^*(C) = e_{j,1}^*(C) = e_1^*(C) = (1+\delta)\frac{\theta_{i,1}\theta_{j,1}}{\left(\theta_{i,1}+\theta_{j,1}\right)^2} R$

(i.c) $e_{i,2}^*(C) = e_{j,2}^*(C) = e_2^*(C) = \frac{\theta_{i,2}\theta_{j,2}}{(\theta_{i,2}+\theta_{j,2})^2} R$

(ii.a) For $k = i, j$, $\sigma_{k,2}^* = y$ if $U_k(\gamma^{S,S}) \geq U_k(\gamma^{S,C})$, while $\sigma_{k,2}^* = n$ otherwise.

(ii.b) If $\sigma^*_{i,2} = \sigma^*_{j,2} = y$, then $\sigma^*_{k,1} = n$ if $U_k(\gamma^{S,S}) \geq U_k(\gamma^{C,\cdot})$, while $\sigma^*_{k,1} = n$ if $U_k(\gamma^{S,S}) < U_k(\gamma^{C,\cdot})$ for $k = i, j$. Otherwise, if $\sigma^*_{l,2} = n$ for some $l = i, j$,then $\sigma^*_{k,1} = y$ if $U_k(\gamma^{S,C}) \geq U_k(\gamma^{C,\cdot})$, while $\sigma^*_{k,1} = n$ if $U_k(\gamma^{S,C}) < U_k(\gamma^{C,\cdot})$ for $k = i, j$.

The proof of this proposition follows directly from the equilibrium values of effort conditional on the outcome of the first stage that we computed previously, and noting that the equilibrium rule followed by the players in the first stage of a period is obtained by comparing the induced equilibrium payoff in the different states. From this proposition, we derive two corollaries. Corollary 4 ranks the different outcomes in terms of total welfare.

Corollary 4. *Suppose that* $(1+\delta)\frac{\theta_{i,1}\theta_{j,1}}{\left(\theta_{i,1}+\theta_{j,1}\right)^2} > \frac{\theta_{i,2}\theta_{j,2}}{(\theta_{i,2}+\theta_{j,2})^2}$*, then* $U_i(\gamma^{S,S}) + U_j(\gamma^{S,S}) > U_i(\gamma^{S,C}) + U_j(\gamma^{S,C}) > U_i(\gamma^{C,\cdot}) + U_j(\gamma^{C,\cdot})$*. If on the other hand,* $(1+\delta)\frac{\theta_{i,1}\theta_{j,1}}{\left(\theta_{i,1}+\theta_{j,1}\right)^2} \leq \frac{\theta_{i,2}\theta_{j,2}}{(\theta_{i,2}+\theta_{j,2})^2}$*, then* $U_i(\gamma^{S,S}) + U_j(\gamma^{S,S}) > U_i(\gamma^{C,\cdot}) + U_j(\gamma^{C,\cdot}) \geq U_i(\gamma^{S,C}) + U_j(\gamma^{S,C})$*.*

Corollary 4 establishes that the socially desirable outcome is a continued settlement since efforts are wasted in contests. It also reveals that there is one condition under which the aggregate effort is lower under an immediate contest than under an delayed one. Two forces matter to determine which outcome generates more aggregate wasteful effort. First, keeping the productivity parameters constant, the marginal benefit of effort is relatively higher in period 1 as it generates a reward for the two periods. As a consequence both players are incentivized to invest more effort in contests occurring in the first time period. This effect is stronger the higher the discount factor. Second, assuming that the players are myopic, the equilibrium value of effort increases as the ratio of productivity parameters converges. Formally, when $\delta = 0$, $e^*_2(C) > e^*_1(C)$ if and only if $\text{Max}\{\frac{e_{i,2}}{e_{j,2}}, \frac{e_{j,2}}{e_{i,2}}\} < \text{Max}\{\frac{e_{i,1}}{e_{j,1}}, \frac{e_{j,1}}{e_{i,1}}\}$. Thus, when the player who is initially more efficient in exerting effort becomes relatively more productive in period 2, the players' aggregate effort higher is greater under an immediate contest than under a delayed contest, for any discount factor. Otherwise, it depends on the magnitude of the discount factor, as it determines the size of the two effects we have just mentioned.

Corollary 5 establishes that if the total effort is higher under an immediate contest than under a delayed one, and the equilibrium strategy of player k in the first stage of period t is to initiate a contest, then other player's equilibrium choice is to settle. If on the other hand the total effort is higher under a delayed contest than under an immediate one, and the equilibrium strategy of a player in the first stage of period 2 is to initiate a contest, then at least one player initiates a contest in period 1.

Corollary 5. *Suppose that* $(1+\delta)\frac{\theta_{i,1}\theta_{j,1}}{\left(\theta_{i,1}+\theta_{j,1}\right)^2} > \frac{\theta_{i,2}\theta_{j,2}}{(\theta_{i,2}+\theta_{j,2})^2}$ *and* $\sigma^*_{k,t} = n$*, then* $\sigma^*_{-k,t} = y$ *for* $k \in \{i,j\}$*,* $t = 1,2$*. If on the other hand,* $(1+\delta)\frac{\theta_{i,1}\theta_{j,1}}{\left(\theta_{i,1}+\theta_{j,1}\right)^2} \leq \frac{\theta_{i,2}\theta_{j,2}}{(\theta_{i,2}+\theta_{j,2})^2}$ *and* $\sigma^*_{k,2} = n$ *for some* $k \in \{i,j\}$*, then* $\sigma^*_{l,1} = n$ *for some* $l \in \{i,j\}$*.*

In period 2, the sum of players' payoff is smaller under a settlement than under a contest. Thus, if one player, say i, has incentives to initiate a contest to modify the allocation of the prize, it follows that the other player is better off in a settlement than in a contest, and thus proposes to settle in equilibrium. In period 1, the argument is analogous. When the efforts wasted are higher under an immediate contest than under a delayed one, the sum of payoffs in an immediate contest is smaller than if both players settle in period 1, whatever the expected outcome in period 2. Thus at most one player may choose to initiate a contest in period 1 at equilibrium. Similarly, if the efforts wasted are higher under a delayed contest than under an immediate one, and a contest is expected to occur in period 2, at most one player may choose to settle in period 1 at equilibrium since the sum of payoffs in an immediate contest is higher than in a delayed one. It follows that a delayed contest cannot occur at equilibrium when the effort wasted is higher under a delayed contest than under an immediate one.

To illustrate our results, we have depicted in Figure 1 the equilibrium first-stage choices of the model when player j's productivity of effort is twice as high as the one of player i ($\theta_{i,1}/\theta_{j,1} = 1/2$), and the players have a unit discount factor ($\delta = 1$). On the horizontal axis, one can read the ratio of effort productivities in period 2 ($\theta_{i,2}/\theta_{j,2}$), and on the vertical axis we have represented the initial share of the prize allocated to player i (α_i). The areas abc and $a^{''}b^{''}c^{''}$ describe the situations where contests are inescapable in period 2 as the ratio of player i over player j's productivity in that period ($\theta_{i,2}/\theta_{j,2}$) is significantly higher or smaller than player i's share of the prize under a status quo (α_i.) In the region abc, player i would initiate a contest with player j in period 2 if a settlement is reached in period 1, and conversely player j would initiate a contest in period 2 in the region $a''b''c''$. In the region $a^{'}b^{'}c^{'}$, on the contrary, players would settle in period 2. When player i would initiate a contest in period 2 (area abc), three outcomes may emerge in period 1. In the zone a, player j initiates a preventive contest. He would be better off would peace prevail in the two periods but as he knows that player i will initiate a contest in period 2, player j is better off by precipitating it in period 1. The incentives of player j to do so increase with the initial share of the prize allocated to player i (α_i), and with the ratio of player i over player j's productivity in period 2 ($\theta_{i,2}/\theta_{j,2}$). In the b area, the two players settle in period 1, even though they are both aware of player i's intention to initiate a contest in period 2. On the one hand, player i prefers to settle in period 1 since he expect to be in a better position to win the contest in period 2. On the other hand, player j is also willing to postpone the contest in order to enjoy his relatively high share of the prize in period 1. Lastly, in the c zone player i initiates a contest in period 1 as he has a small initial share of the prize and he does not expect a strong increase in the relative productivity of his effort in the future. Following the same reasoning, in the zones a' and a'' (c' and c'') player j (i) initiates a contest in period 1, whilethe players settle in b' and in b''. In b', they settle in period 1 expecting settlement in period 2, while in b'' the players expects a future contest, and both find it optimal not to precipitate it in period 1.

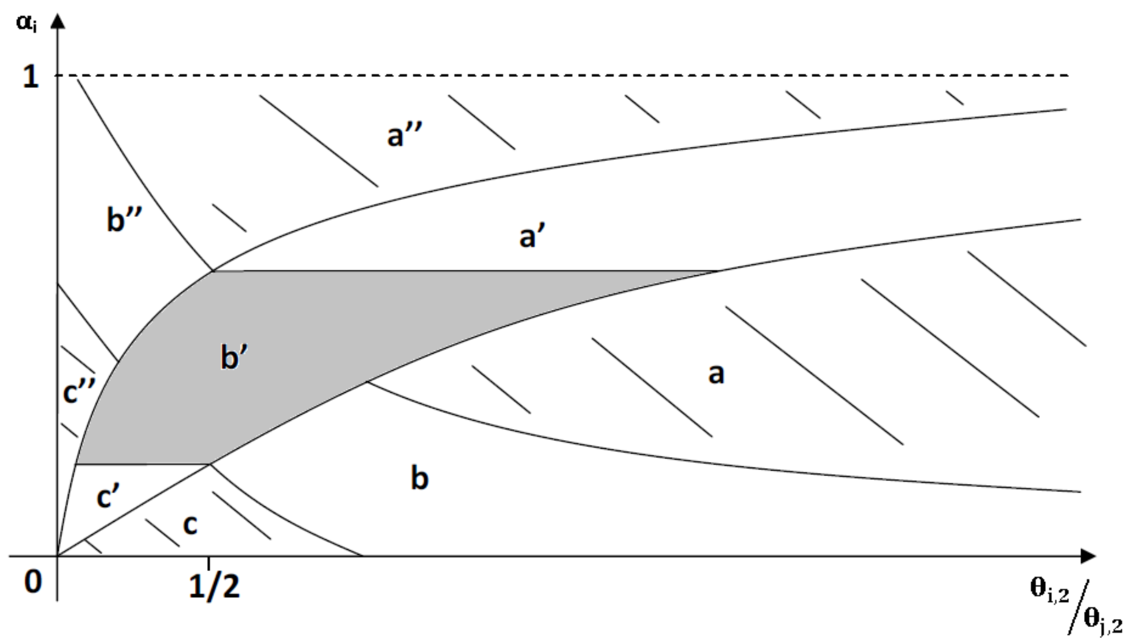

Figure 1: Outcomes for $\alpha_1 = 0.5$ and $\delta = 1$

Lemma 6 establishes that when a contest is initiated, the more efficient is a player in exerting effort relative to the other, the higher is his equilibrium payoff.

Lemma 6. *We have (i)* $\partial U_k(\gamma^{S,C})/\partial(\theta_{k,2}/\theta_{-k,2}) > 0$*, and (ii)* $\partial U_k(\gamma^{C,\cdot})/\partial(\theta_{k,1}/\theta_{-k,1}) > 0$*.*

Proof. Rewriting $U_k(\gamma^{S,C})$ as $\left(\alpha_k + \delta \frac{(\theta_{k,2}/\theta_{-k,2})^2}{(\theta_{k,2}/\theta_{-k,2}+1)^2}\right) W$ and differentiating w.r.t. $\theta_{k,2}/\theta_{-k,2}$ yields (i). Similarly, $U_k(\gamma^{C,\cdot})$ can be written as $(1+\delta)\frac{(\theta_{k,1}/\theta_{-k,1})^2}{(\theta_{k,1}/\theta_{-k,1}+1)^2} W$, and we therefore easily deduce (ii). ■

In Proposition 7, we show that there is a non-monotonic relationship between the discount rate and the outcome of the first stage of period 1.

Proposition 7. *Let* $\theta_{i,2}/\theta_{j,2} > \theta_{i,1}/\theta_{j,1} \geq 1$*. Suppose that* $\alpha_i W < \frac{\theta_{i,1}^2 W}{(\theta_{i,1}+\theta_{j,1})_{k,2}^2}$*. Then,*

(i) $(\sigma_{i,2}^*, \sigma_{j2}^*) = (n, y)$*,*

(ii) there exists $\underline{\delta}$ *satisfying* $\underline{\delta}> 0$ *such that* $(\sigma_{i,1}^*, \sigma_{j,1}^*) = (n, y)$ *for* $0 \leq \delta <\underline{\delta}$*,*

(iii) there exists $\overline{\delta}$ *where* $\overline{\delta} > \underline{\delta}$ *such that* $(\sigma_{i,1}^*, \sigma_{j,1}^*) = (y, y)$ *for* $\underline{\delta}\leq \delta < \overline{\delta}$*, while* $(\sigma_{i,1}^*, \sigma_{j,1}^*) = (y, n)$ *for* $\delta > \overline{\delta}$*.*

Proof. Let $\Delta_k = U_k(\gamma^{C,\cdot}) - U_k(\gamma^{S,C}) = W(\frac{\theta_{k,1}^2}{\left(\theta_{k,1}+\theta_{-k,1}^2\right)^2} - \alpha_k) + \delta W(\frac{\theta_{k,1}^2}{\left(\theta_{k,1}+\theta_{-k,1}^2\right)^2} - \frac{\theta_{k,2}^2}{\left(\theta_{k,2}+\theta_{-k,2}^2\right)^2})$. Notice first that for every $\delta \geq 0$, the condition $\theta_{i,2}/\theta_{j,2} > \theta_{i,1}/\theta_{j,1} \geq 1$ implies that the surplus is higher when the contest occurs in period 2 than in period 1 (see Corollary 4). Thus, $\Delta_i + \Delta_j < 0$.

(i) Notice that if we had $\theta_{i,2}/\theta_{j,2} = \theta_{i,1}/\theta_{j,1}$, we would have obtained $\sigma_{i,2}^* = n$ since $\alpha_i W < \frac{\theta_{i,1}^2 W}{(\theta_{i,1}+\theta_{j,1})_{k,2}^2}$. Since $\theta_{i,2}/\theta_{j,2} > \theta_{i,1}/\theta_{j,1}$, we have $\sigma_{i,2}^* = n$ by Lemma 6. By Corollary 5, we then have $\sigma_{i,2}^* = y$.

(ii) Let $\delta = 0$. Then we have that $\Delta_i > 0$ since $\alpha_i W < \frac{\theta_{i,1}^2 W}{(\theta_{i,1}+\theta_{j,1})_{k,2}^2}$, which implies that $\sigma_{i,1}^* = n$. It follows that $\Delta_j < -\Delta_i < 0$, and thus $\sigma_{j,1}^* = y$. One can see that Δ_i is strictly decreasing in δ.[4] Thus, there exists a threshold $\underline{\delta}> 0$ such that for all $\delta <\underline{\delta}$, $\Delta_i > 0$, implying that $\sigma_{i,1}^* = n$. and $\sigma_{j,1}^* = y$, while $\Delta_i < 0$ for $\delta > \underline{\delta}$.

(iii) When $\delta =\underline{\delta}$, $\Delta_i = 0$, implying that $\Delta_j < 0$, and $\sigma_{i,1}^* = \sigma_{j,1}^* = y$. As Δ_j is strictly increasing in δ,there is a threshold $\overline{\delta}$ such that for all $\delta < \overline{\delta}$, $\Delta_j < 0$ and $\sigma_{j,1}^* = y$, while for $\delta > \overline{\delta}$, $\Delta_j > 0$ and $\sigma_{j,1}^* = n$. ■

Proposition 7 reveals the non-monotonic relationship between the discount rate and the incentives of players to achieve short settlements despite the prospects of future contests. To grasp the essence of the proposition, it is worth reasoning in terms of instantaneous utilities. Assuming that $\alpha_2 > \alpha_1$ implies that the instantaneous incentives of i to initiate a contest in period 2 are higher

[4]The derivative of Δ_k with respect to δ is $\left(\frac{\theta_{k,1}^2}{\left(\theta_{k,1}+\theta_{-k,1}^2\right)^2} - \frac{\theta_{k,2}^2}{\left(\theta_{k,2}+\theta_{-k,2}^2\right)^2}\right) W$.

than in period 1. Consider further that when no weight is assigned to the future ($\delta = 0$) player i is willing to provoke a contest in period 1. Since the instantaneous profitability of a contest is higher in period 2 as compared to period 1, increasing the weight given to the future incentivizes player i to agree on delaying the contest. For player j, however, the situation is the opposite since this player would prefer reaching a settled solution in both periods, while his instantaneous utility of contest is higher in period 1 as compared to period 2. Conditional on contest being inescapable because of player i initiating the contest in period 1 ($\delta < \underline{\delta}$) or in period 2 ($\delta \geq \underline{\delta}$), increasing the discount rate will therefore incentivize player 2 to preventively provoke a contest in period 1 so as to avoid a particularly disadvantageous contest in period 2. In the proof of Proposition 7 we demonstrate that there will always exist a range of δ values such that (i) the discount rate is sufficiently high for player i to prefer postponing the contest because of his dynamic increase in strength, and (ii) the discount rate is sufficiently low for player j not to want to preventively initiate a contest in period 1.

Provided $\bar{\bar{\delta}} < 1$, we have therefore established that the relationship between the discount rate and contest is $U - shaped$, since there exists an intermediate range of discount rates ($\delta \in [\underline{\delta}, \bar{\bar{\delta}}[$) for which the players will agree to reach short-lived settlements. It is noteworthy that while is it possible that $\bar{\bar{\delta}} < 1$, neither $\bar{\bar{\delta}}$, nor $\underline{\delta}$ have been shown to always lie in the zone of admissible parameters (i.e. $[0, 1]$). Indeed, we may encounter parameters' configurations such that $\underline{\delta} > 1$, in which case player i will always prefer going to initiate a contest in period 1, or even $\underline{\delta} < 1 < \bar{\bar{\delta}}$, in which case player j will never preventively provoke a contest.

Our model therefore accommodates scenarii that corroborate the findings of Skaperdas and Syropoulos (1996), Skaperdas (2006), and McBride and Skaperdas (2007) since for $\delta > \underline{\delta}$ higher discount rates make contests more likely. On the other hand, higher discount rates favour settlements for $\delta < \underline{\delta}$ since they reduce the incentives of the more contest-prone player to renounce on the settled solution in the short run.

5 Discussion

Our two-players two-stage contest model features a series of assumptions that are worth discussing for better grasping the reach of our conclusions.

Commitment

The chosen timing whereby players first decide whether or not to enter a contest and then decide the effort level is tantamount to assuming a strong commitment capacity on behalf of the players since we rule out the possibility of reneging on the first stage decision. From a technical point of view, this assumption is necessary for settlements to ever arise in our setting. Indeed, with a reverted timing where the effort decisions would occur in the first stage, the contest being modeled as a zero-sum game, the players would at best be simultaneously indifferent between playing y and n, with no configuration making both players opt for settlement. From a conceptual viewpoint, this commitment capacity may equally be interpreted as players facing important reputation costs of violating their first stage decision, or the players' effort slowly translating in contest capacity so that the opponent is never subject to a sneak attack. Lastly, from a practical viewpoint, contests such as legal or sports contests whereby the players ought to publicly declare their participation before the contest actually taking place satisfy the model's timing assumptions. Our setting equally applies to military confrontations since they necessitate military preparation which is likely to be observed by one's opponent. Lastly, our model is probably less relevant for understanding lobbying activities or R&D research since in these cases the potential contestants can secretly invest effort into a forecasted contest.

Accumulation

We assume that the players effort maps into instantaneous capacity to improve their odds in the contest, which amounts to assuming a total depreciation of the players' contest effort. Given, however, that the both players' payoff and the production technology are linear, accommodating for effort accumulation would not alter the picture since the players would have no advantage in investing effort in period 1 instead of deferring this action to period 2 where the cost is discounted at some rate δ. If either costs or utility were non-linear, however, introducing effort accumulation would partially modify the results since players would seek to smooth their consumption and expenditures through time. This would reduce the scope for reaching settled solutions in the short run since players would anyway invest some resources in contest effort in period 1.

"Win" *vs* "share" contests

We have chosen to adopt the "win" contests approach to make the model comparable to a specific strand of the literature. Yet, a series of papers consider the alternative "share" contests approach whereby no contestant is eliminated from future confrontations. This distinction is certainly fundamental in establishing our results. Keeping in mind the findings of Proposition 2, notice that in a "share" contest, and in the absence of effort accumulation, the subgames of settlement or contest in period 1 are likely not to drastically differ. As a consequence any reduction in future costs, or any change in the discount factor is unlikely to radically affect the players' incentives to initiate a contest in period 1.

6 Conclusion

In this paper we propose a comprehensive and unifying framework for the study of two-period contest games featuring two players. We establish two sufficient conditions for players to agree on short run settled solutions despite them forecasting a contest in the future. These two conditions are that either players at least partially discount the future, or that the costs associated with the contest be lower in the future for both players. The future cost reduction may result from various mechanisms. When endogenizing the players' contest effort choice we unveil one particular such mechanism: if the relative contest productivity of the more efficient contestant increases through time, the players' aggregate contest effort is lower in future confrontations, thus creating scope for short run settlements.

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