Energy, Complexity and Sustainable Long-term Growth

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Abstract

We introduce the concept of product complexity in an endogenous growth model with renewable energy and expanding product variety *à la* Grossman-Helpman (1991). We describe the complexity of a product as an increasing function of the variety of inputs it consists of. Considering that energy is necessary to all human activities (including research), we highlight what type of long run growth path is possible according to a) the potential of energy efficiency gains in the various human activities and b) the effect of the product complexity on the energy intensiveness of its production process. In a finite world, a neoclassical growth path where economic growth can be both quantitative and non-quantitative (i.e. takes the form of an increase in the quantity of produced goods and in the product variety) is only possible if the potential of energy efficiency gains is unbounded in all human activities. If the energy intensiveness of the final production is bounded from below by a strictly positive constant, quantitative growth is not possible in the long run but non-quantitative growth may persist if (i) the impact of product complexity on the energy intensiveness of production is null or weak enough *and* (ii) the energy intensiveness of research activities tends to zero. If these conditions are not met, no form of long run growth is possible.

Keywords: product complexity, energy, long-term growth.

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1 Introduction

Since the controversial book of Meadows and al. (2004 for the update of the 1972 edition), the existence of physical limits to growth in a finite world has remained a debated issue, in particular between economists. On the optimists' side, the advocates of the "weak sustainability" postulate consider that technical progress and substitution possibilities between natural resources and man-made capital will make long-term growth possible. More pessimistically, ecological economists view the "weak sustainability" postulate as wishful thinking: they argue that fundamental physical laws such as the principles of thermodynamics limit both the substitution possibilities between natural and man-made inputs and the potential of resource efficiency gains that can be expected from technical progress. For them, "forever" growth is thus impossible in a finite world.

In theoretical models as in real life, economic growth is multiform. Growth may be purely quantitative and take the form of an increase in the quantity of produced goods and services, like in the canonical one-sector growth model; but it may also be non-quantitative and take the form of an expanding product variety or of a rising product quality (see e.g. chapters 3 and 4 of Grossman-Helpman, 1991). The debate on the limits to growth has often taken place without distinguishing explicitly the quantitative and non-quantitative dimensions of growth. Such a distinction may however matter because the non-quantitative dimension of growth might not be constrained by physical laws to the same extent as the quantitative one. For instance, in a model with rising product quality and with a finite but recyclable essential resource, Fagnart and Germain (2011) show that the principle of matter conservation makes a perpetual quantitative growth impossible but lets a purely qualitative growth possible in the long run. This paper however ignores any energy issue although it might influence the debate. Jeanmart and Possoz (2013) argue -without showing it formally- that a long-term path with a purely qualitative growth is only possible if the rising quality of human productions is not accompanied by an increasing complexity of these productions and/or of their production processes. Their argument relies on thermodynamics which establishes a positive relationship between the complexity of a system and its energy requirements: if rising quality implied rising complexity, a growth process based on quality improvements would get increasingly energy consuming and all forms of economic growth might thus come to a halt in a finite world.

The present article introduces the concept of product complexity into an endogenous growth model with expanding input variety à la Grossman-Helpman (1991, Chapter 3). It explores what type of long run growth path is possible according to a) the potential of energy efficiency gains in the various human activities and b) the link between complexity and energy intensiveness of final productions. The expanding variety model offers a framework in which the complexity of a product may be linked to its composition in a simple and intuitive way: a product is more complex when it consists of a larger number of different inputs. So defined, our concept of complexity is analogous to the one encountered in several disciplines which link the complexity of a system to the number and variety of its parts or components. In order to remain close to the subject of the present paper, we shall only make reference to contributions interested in the complexity of production processes. In Economics, two dimensions of their complexity have been put forward[1]: the complexity of the organizational structure in which production operations take place (e.g. in Becker et Murphy, 1992) and, as in the present paper, the complexity of the product itself (e.g. in Kremer, 1993). Becker and Murphy consider a production process consisting of a given number of complementary tasks. Division of labour (i.e. the specialization of teams of workers on subsets of tasks) raises labour productivity according to the well-known Smithian argument. But it complexifies simultaneously the organization of the production process by requiring to coordinate complementary workers' teams. A more extensive division of labour thus makes the organization more complex and increases the costs of coordination, communication and supervision[2]. In his O-ring theory of development, Kremer (1993) links the complexity of a product to the number

[1] Note that in Economics, the concept of complexity is also used in the sense of dynamic complexity, i.e. the complexity of the possible dynamic behaviours of a model (e.g. chaotic dynamics). See e.g. Day (2009).

[2] The complexity of the organizational design of a firm is further investigated in Garicano (2000).

of tasks necessary to its production: a larger number of tasks makes the product more valuable but also more complex. Moreover, a production requiring more tasks is characterized by more potential areas for human mistakes that could ruin the value of the product. In order to reduce this risk, a firm that produces a more complex product must employ higher skilled workers and pay higher wages. In Becker-Murphy as in Kremer, complexity (of the organization in the former, of the product in the latter) thus brings benefits and costs and is expected to increase with economic development[3].

In the field of manufacturing systems engeneering, papers like Fujimoto *et alii* (2004), Hu *et alii* (2008) or Modrak and Marton (2013) describe the complexity of production operations in a firm as a function of the variety of products, the variety of their functions and the variety of intermediary inputs and equipments. This literature constructs complexity indices based on the computation of the informational entropy of production operations, which depends on the quantity of information necessary to describe theses operations. Though highly stylized, our concept of product complexity is consistent with what is done in this literature or in management science (see e.g. Novak and Eppinger, 2001).

In all the above mentionned references, the cost of complexity is acknowledged but not expressed in terms of energy consumption. The step between the two is however easily taken if one considers that all the activities that managing complexity requires (from coordination to information communication/processing and input manipulation) are *in fine* energy consuming. Moreover, thermodynamics (as already mentionned) and information theory invite to link positively complexity and energy[4,5]. Johansson (2002) makes such a link. In terms analogous to those of thermodynamics, he defines the complexity of a product as the number of possible combinations of the elements necessary to its production. He puts forward an increasing mathematical relationship between the energy content of a product and its complexity measured by the logarithm of the number of its functional components. Let us mention in passing that a positive relationship beween complexity and energy is also put forward by the anthropologist Tainter (see e.g. Tainter, 2011) who speaks of an "energy-complexity spiral" to describe that a positive relationship exists between the evolution of the organizational complexity of a society and the one of its energy requirements.

In this short article, we propose a model in which all human activities (production of goods and services, product and technological innovations,...) require energy and we explore the implications of the energy requirements of these activities on long-term growth possibilities. We do not aim at "micro-founding" the link between complexity and energy: we admit that a relationship between the two may exist for the reasons explained hereabove but do not impose any *a priori* restriction on this relationship. By doing so, we can examine how the properties of this relationship affect the type of possible long-term growth. Section 2 presents an expanding product variety model *à la* Grossman-Helpman (1991, Chapter 3) where we reinterpret the fixed primary factor of production as renewable energy (instead of labour in the original model). The final good is the output of the assembly process of a variety of man-made inputs and its complexity is increasing in the input variety used in the process. In section 3, we identify three possible types of long-term growth path according to the potential of energy efficiency gains in the various human activities and to the presence and strength of the complexity effect: a mixed ("quantitative" *and* "non quantitative") growth path, a purely "non-quantitative" path and a zero growth path. Section 4 concludes.

[3]In Becker and Murphy, knowledge accumulation and economic development raise the optimal level of division of labour and thereby the complexity of the production organization. In Kremer, a development accompanied by an increase in workers' skills leads an economy to specialize in more complicated products.

[4]According to the Second Law of Thermodynamics, any isolated system that is let to itself tends toward the state of maximum disorder or entropy. This maximum level of entropy is a measure (roughly speaking, the logarithm) of the number of ways in which the system may be arranged. In order to increase the order of the system (or to decrease its entropy) to a certain level, energy must be brought to the system from outside and the necessary quantity of energy is increasing in the maximum level of entropy of the system (in other words, in its complexity). A product is a system made of its parts and, by analogy, making it more complex may thus require more energy.

[5]In information theory, the complexity of a system is increasing in its informational content, i.e. in the amount of information necessary to describe it. As the processing of a bit of information has an energy content, there is here again a positive relationship between complexity and energy requirements.

2 The model

There are two main activities: production and research. Production consists of two successive steps of operations. In a first step, different elementary goods or "components" (inputs in the sequel) are processed. In a second step, these inputs are assembled into a final good (FG hereafter).

Research activities allow the economy to create deterministically new varieties of inputs. But they also enlarge the public knowledge capital and so generate technological progress as a by-product: the research efforts of a period reduce the energy intensiveness of production and research activities in the next period.

The economy enjoys a constant flow of renewable energy (say solar energy)[6]. Energy is necessary to all operations of production as well as research. We thus consider energy as the "ultimate" resource on which depend all human activities (including human work) and their outputs (including productive equipment): see e.g. Ayres and Warr (2009). For the sake of brevity, labour and physical capital are not explicitly present in the model. However, our conclusions linking the possible types of long run growth to the potential of energy efficiency gains would remain qualitatively the same in a model with energy, labour and capital.

Since we only aim at identifying (the feasibility of) different *classes* of long-term solutions and do not want to derive precise trajectories, we do not need neither to make explicit the institutional organisation of the economy (e.g. whether it is decentralised or not) nor to model *all* economic decisions. As a consequence, we do not need to distinguish particular agents (firms, households or a central planner) and we only speak of "the economy".

2.1 Production

2.1.1 Intermediate and final goods and technologies

There is a variety of intermediate inputs represented by the interval $[0, n_t]$. Variable n_t is thus the measure (or "number") of inputs that have been invented until the beginning of period t. It is possibly increasing through time. An homogenous final good (FG) is produced by assembling the inputs. Without loss of generality, let $[0, m_t]$ (with $m_t \leq n_t$) be the subset of inputs effectively used in t. The assembly process of quantities y_{it} of each input $i \in [0, m_t]$ into a quantity F_t of FG is described by the following technology:

$$F_t = \mathcal{F}\left(\{y_{it}\}_{0 \leq i \leq m_t}, m_t\right), \tag{1}$$

where functional $\mathcal{F}$ is positive, monotonically increasing, symmetric and homogenous of degree 1 in $\{y_{it}\}_{i \in [0,m_t]}$[7]. The fact that m_t is an argument of $\mathcal{F}$ describes a positive variety effect in the FG production process (see also next subsection). A well-known example of $\mathcal{F}$ is the functional *à la* Dixit-Stiglitz given by

$$F_t = \left[\int_0^{m_t} y_{it}^{\alpha} di\right]^{1/\alpha}, \qquad \text{where } 0 < \alpha < 1. \tag{2}$$

Producing a quantity y_{it} of input $i \in [0, n_t]$ requires a quantity of energy $\mu_t y_{it}$, where $\mu_t > 0$ measures the energy intensity of the process[8]. By aggregating the energy requirements of input

[6] By definition, non-renewable energy sources are depletable and can only play a transitory role. Because long-term growth can only rely on renewable resources and because we only focus on long-term issues, we disregard the transitory era where non-renewable energy sources are in use.

[7] This formulation is close to the one in Benassy (1996) and only departs from it because (i) the set of inputs is continuous and (ii) m_t is here a choice variable.

[8] For the simplicity of the exposition, we assume that μ_t is identical for all i's but this assumption is not at all crucial to our results.

productions, one writes the total energy consumed by the input sector in period t as

$$E_{pt} = \int_0^{m_t} \mu_t y_{it} di = \mu_t \int_0^{m_t} y_{it} di. \tag{3}$$

The energy intensity μ_t may decrease through time as a result of technological progress (see subsection 2.2).

FG production also requires energy: the quantity of energy necessary to the assembly process of inputs $\{y_{it}\}_{0 \le i \le m_t}$ is given by:

$$E_{at} = \mathcal{E}\left(\{y_{it}\}_{0 \le i \le m_t}, m_t\right), \tag{4}$$

where functional $\mathcal{E}$ is a positive, monotonically increasing, symmetric and homogenous of degree 1 in $\{y_{it}\}_{0 \le i \le m_t}$. The positive relationship between E_{at} and $\{y_{it}\}_{0 \le i \le m_t}$ means that assembling more units of inputs consumes more energy. Any input manipulation or transformation is indeed an operation that requires energy. So, the energy requirements of the final good production is increasing in the number of such operations, i.e., in the number of units of inputs to manipulate and assembly. Functional $\mathcal{E}$ may also depend on the variety of inputs used in the process, which describes the link between complexity and energy intensiveness of the assembly process. We further describe this point in the next section.

2.1.2 The symetric case

As functionals $\mathcal{F}$ and $\mathcal{E}$ are symetric in $\{y_{it}\}_{0 \le i \le m_t}$, production plans are such that all inputs are used in equal quantities, i.e. $y_{it} = y_t, \forall i \in [0, m_t]$. Following Benassy (1996), one may then rewrite $\mathcal{F}\left(\{y_t\}_{0 \le i \le m_t}, m_t\right) = f(y_t, m_t)$, where f is homogenous of degree 1 in y_t and increasing in its two arguments. Hence,

$$F_t = f(1, m_t)\, y_t = \upsilon(m_t)\, m_t\, y_t, \quad \text{where } \upsilon(m_t) =_{def} \frac{f(1, m_t)}{m_t}. \tag{5}$$

F_t appears as a composite index with two dimensions: (i) a *quantitative* dimension proportional to the number of input units $m_t y_t$ involved in the assembly process and (ii) a *non-quantitative* dimension linked to a "*variety*" effect measured by the term $\upsilon(m_t)$. This variety effect is an increasing function of m_t if the elasticity of $f(1, m_t)$ with respect to m_t is larger than 1, what we will assume. In the Dixit-Stiglitz case (2), $\upsilon(m_t) = m_t^\beta$ with $\beta = \frac{1}{\alpha} - 1 > 0$ (since $\alpha < 1$): a higher value of parameter β means a bigger variety effect.

Let Y_t be the quantity index corresponding to the number of input units used in the assembly process:

$$Y_t = m_t y_t. \tag{6}$$

(5) then writes as $F_t = \upsilon(m_t) Y_t$.

Similarly, one may rewrite E_{at} as $E_{at} = \mathcal{E}\left(\{y_t\}_{0 \le i \le m_t}, m_t\right) = e_a(y_t, m_t)$, where e_a is increasing and homogenous of degree 1 in y_t. Hence,

$$E_{at} = e_a(1, m_t)\, y_t = \lambda(m_t) Y_t, \text{ where } \lambda(m_t) =_{def} \frac{e_a(1, m_t)}{m_t}. \tag{7}$$

The energy requirement of the assembly process thus depends both on the processed quantity of inputs ($Y_t = m_t\, y_t$) and on the number of input types m_t via the term $\lambda(m_t)$. Function $\lambda(m_t)$ captures the effect of the product complexity on the energy intensiveness of its manufacturing. If $\lambda(m_t)$ is a constant (i.e. if m_t is not an argument of function $\mathcal{E}$), the energy intensiveness of the production process depends on the number of inputs to assembly Y_t but not on the number of input types: assembling m different inputs (each in unit amount) is neither more nor less energy consuming than assembling m units of a single input. If $\lambda(m_t)$ is increasing in m_t (i.e. if function $\mathcal{E}$ is increasing in m_t), manufacturing a more complex product (i.e. a product consisting of a larger number of input types) is more energy consuming than manufacturing a simpler one, made with the same number of input units Y_t but with a smaller number of input types. The reverse is true if $\lambda'(m_t) < 0$.

In summary, if the tasks necessary to the production of a more complex product are energy consuming, $\lambda'(m_t) > 0$. In order to highlight the importance of this complexity effect on the feasibility of growth, we shall also consider the case where it is absent (i.e. where $\lambda'(m_t) \leq 0$).

Given (3) and (7), the total energy necessary to produce F_t units of final good (i.e. the production and assembly of $Y_t = m_t y_t$ units of inputs), is given by

$$E_{ft} = [\mu_t + \lambda(m_t)]\ Y_t. \tag{8}$$

Together (5) and (8) imply:

$$F_t = \frac{\upsilon(m_t)}{\mu_t + \lambda(m_t)} E_{ft}. \tag{9}$$

The energy intensiveness of final production (i.e. the ratio E_{ft}/F_t) thus depends positively on the energy intensiveness of input and FG productions ($\mu_t + \lambda(m_t)$) and negatively on the "variety" effect ($\upsilon(m_t)$).

2.1.3 Choice of m_t in a given time period

Since we focus on feasibility issues, we are only interested in production plans that 1) do not imply any resource waste and 2) lie on the frontier of feasible values of F_t in a given period. We thus consider the variety m_t that maximises final good production F_t, given the n_t existing input types and the quantity of energy available for production E_{ft}. For a given E_{ft} and given (9), this value of m_t is the solution to the following problem:

$$\max_{0 \leq m_t \leq n_t} \ F_t = \frac{\upsilon(m_t)}{[\mu_t + \lambda(m_t)]} E_{ft}. \tag{10}$$

The first derivative of F_t with respect to m_t is proportional to the following expression[9]:

$$\frac{\partial F_t}{\partial m_t} \quad \sim \quad \upsilon'(m_t)\,[\mu_t + \lambda(m_t)] - \upsilon(m_t)\lambda'(m_t). \tag{11}$$

An increase in m_t has thus two effects on F_t at given E_{ft}. The first term at the right-hand-side of (11) captures the variety effect which is unambiguously positive. The other effect follows from the impact of m_t on the energy intensiveness of the final good production process, impact which depends on the sign of $\lambda'(\cdot)$:

1. If an increase in the variety of inputs m_t used in the assembly process decreases (or leaves unchanged) the energy intensiveness of this process (i.e. if $\lambda'(.) \leq 0$), (11) is always strictly positive and the solution to (10) is $m_t = n_t$. Intuitively enough, if product complexity brings benefits but induces no cost, all the existing input types will be used[10].

2. If complexity is energy consuming (i.e. if $\lambda'(.) > 0$), m_t has two opposite effects on F_t and there may exist an interior maximum $\bar{n}_t$ of problem (10). Such a $\bar{n}_t$ must be the solution of the first order condition

$$\frac{\upsilon'(\bar{n}_t)}{\upsilon(\bar{n}_t)} = \frac{\lambda'(\bar{n}_t)}{\mu_t + \lambda(\bar{n}_t)}. \tag{12}$$

[9]We use the notation $a \sim b$ to mean that a is proportional to b.

[10]This corresponds to the "standard" solution of Grossman-Helpman (1991, chapter 3) and Benassy (1996).

In words, when increasing m_t makes final production more energy intensive, the optimal $\bar{n}_t$ is the value of m_t that equalizes the marginal variety effect of an increase in m_t to the marginal energy cost of this increase. This $\bar{n}_t$ will be an interior maximum only if $\bar{n}_t < n_t$ and if the objective is concave at $\overline{n}_t$. This requires that

$$\upsilon''(\overline{n}_t)\left[\mu_t + \lambda(\overline{n}_t)\right] - \upsilon(\overline{n}_t)\lambda''(\overline{n}_t) < 0. \tag{13}$$

A sufficient but not necessary condition for this inequality to hold is (i) a concave variety effect $\upsilon(.)$ and (ii) a convex complexity effect $\lambda(.)$.

2.2 Research activities and technological spillovers

Research activities allow the economy to invent new input types. This process is deterministic and consumes energy. The energy consumption of research activities in t, E_{rt}, is assumed proportional to the research effort measured by the number of new inputs invented during the period:

$$E_{rt} = \varepsilon_t \left[n_{t+1} - n_t\right]. \tag{14}$$

$\varepsilon_t(\geq 0)$ measures the energy required to invent a new type of input. Because $E_{rt} \geq 0$, the inequality $n_{t+1} \geq n_t$ holds necessarily. This formulation thus assumes (as in Grossman-Helpman, ch. 3) that there is no input obsolescence: once created, a given input variety remains available forever.

Moreover, we assume that research made to increase n_t enlarges the “stock” of public knowledge. This feeds in turn a process of technological progress which diffuses in the economy and improves the energy efficiency of production and research activities. Following Grossman-Helpman (1991, section 3.2), public knowledge in t is measured by n_t, and we suppose

$$\mu_t = \mu(n_t) \quad \text{and} \quad \varepsilon_t = \varepsilon(n_t), \qquad \text{with } \mu'(.) \text{ and } \varepsilon'(.) \leq 0, \tag{15}$$

i.e. technological progress makes input production and research activities less energy intensive.

2.3 Frontier of the possible production and research activities

The economy enjoys a constant flow of energy E, which can be freely captured. This optimistic assumption is made in the perspective of identifying the frontier of production and research activities. Along this frontier, E is always fully used and shared between production and research: $E = E_{ft} + E_{rt}$. Given (8) and (14), this implies that

$$E = \left[\mu(n_t) + \lambda(m_t)\right] Y_t + \varepsilon(n_t)\left[n_{t+1} - n_t\right], \tag{16}$$

where $Y_t = m_t\, y_t$.

In period t, the frontier of feasible combinations of production and research is thus given by (16) and

$$F_t = \upsilon(m_t) Y_t \tag{17}$$

$$m_t = \begin{cases} n_t & \text{if} \quad \lambda'(\cdot) \leq 0 \\ \min\{\bar{n}_t, n_t\} & \text{if} \quad \lambda'(\cdot) > 0, \end{cases} \tag{18}$$

where $\bar{n}_t$ is given by (12).

3 Possible long-term trajectories

We analyse now the possibility of long-term growth, i.e. the feasibility of long-term trajectories with non-decreasing final output F_t. In this model where final output growth may be non-quantitative as well as quantitative, we identify what type of long-term growth is sustainable according to the long-term properties of energy intensities $\mu(\cdot)$ and $\varepsilon(\cdot)$ and complexity effect $\lambda(\cdot)$. We structure this discussion in function of the sign of $\lambda'(m_t)$. Although this analysis could be held in more general terms, it is more reader-friendly to assume the following isoelastic functional forms for the variety and complexity effects:

$$\upsilon(m_t) = m_t^{\beta}, \tag{19}$$

$$\lambda(m_t) = c_\lambda m_t^{\bar{\lambda}}, \tag{20}$$

where c_λ is a strictly positive constant. As mentioned in the discussion following (5), (19) corresponds to the expression of $\upsilon(m_t)$ in the Dixit-Stiglitz case (2) and $\beta > 0$. According to the sign of parameter $\bar{\lambda}$, complexity is energy consuming (if $\bar{\lambda} > 0$) or not (if $\bar{\lambda} \leq 0$).

Under assumptions (19) and (20), the derivative (11) is always strictly positive if $\bar{\lambda} \leq \beta$ but admits a unique root if $\bar{\lambda} > \beta$. Hence, (18) becomes:

$$m_t = \begin{cases} n_t & \text{if} \quad \bar{\lambda} \leq \beta \\ \min\{\bar{n}_t, n_t\} & \text{if} \quad \bar{\lambda} > \beta, \end{cases} \tag{21}$$

where $\bar{n}_t$ is the unique solution of (12): under assumptions (19) and (20),

$$\overline{n}_t = \left[\frac{\beta}{\bar{\lambda} - \beta} \frac{\mu(n_t)}{c_\lambda} \right]^{\frac{1}{\bar{\lambda}}}. \tag{22}$$

It is decreasing in n_t.

Hereafter, we define a balanced growth path (BGP) as a long-term trajectory with (i) a constant and strictly positive growth rate of F_t, (ii) constant positive growth rates of variables m_t and n_t, (iii) constant (possibly negative) growth rate of y_t and (iv) constant shares of energy allocated to production and research (constant ratios E_{jt}/E, $j = f, r$ or equivalently constant flows E_{jt}, $j = f, r$).

Given (15) and property (iv) of a BGP, (14) may be rewritten as follows along a BGP:

$$E_r = \varepsilon(n_t) n_t \left[\frac{n_{t+1}}{n_t} - 1 \right].$$

Since property (ii) of a BGP implies a constant growth rate of n, the above equation implies that the product $\varepsilon(n_t)n_t$ must be constant (say $c_\varepsilon > 0$) along a BGP, i.e.

$$\varepsilon(n_t) = c_\varepsilon / n_t. \tag{23}$$

Furthermore, when $m_t = n_t$ (which is certainly the case when $\bar{\lambda} \leq \beta$ (see (21)), (8) and (15) can be rewritten as

$$Y_t = \frac{E_f}{\mu(n_t) + c_\lambda n_t^{\bar{\lambda}}} \tag{24}$$

$$F_t = n_t^{\beta} Y_t. \tag{25}$$

Let us analyse now the different possible cases according to the value of $\bar{\lambda}$.

3.1 No positive effect of complexity on energy consumption

3.1.1 $\lambda(m)$ is strictly decreasing, i.e $\bar{\lambda} < 0$

If the elasticity of $\lambda(.)$ is strictly negative, a BGP characterised by the positive growth of the *two* dimensions of F_t is possible: economic growth may be both (i) *quantitative* through the increase of Y_t and (ii) *non quantitative* through the increase of the variety n_t. As neoclassical growth theory has (more or less explicitly) a very broad interpretation of what economic growth can be, we label neoclassical a BGP along which "all forms" of growth are possible in the long run.

Proposition 1 *In an economy endowed with a renewable but finite energy flow, a neoclassical BGP (i.e. a BGP with a strictly positive growth of F_t, m_t and Y_t) is only possible if the potential of energy efficiency gains in all human activities (production and research) is unbounded:*

1. *Quantitative growth (i.e. growth of Y_t) requires that the energy intensity of intermediary and final productions, $\mu_t + \lambda_t$, tends to zero when $n_t \to +\infty$.*
2. *Non-quantitative growth (i.e. growth of n_t (or m_t)) requires that the energy intensity of research activities, ε_t, tends to zero as $n_t \to +\infty$.*

Point 1 follows straightforwardly from (24): the denominator of its right-hand-side must tend toward zero for a perpetual growth of Y_t to be possible with a constant E_f. Point 2 is a direct consequence of (23).

The above proposition makes explicit a key assumption on which relies the weak sustainability postulate. In a finite world, quantitative growth could only be sustained in the long run if the energy content of a unit of final output tended progressively toward zero. Even though some might argue that research activities are more immaterial and less energy intensive than the production of material goods, note that the feasibility condition of a long-run non-quantitative growth is also very strong : Point 2 means indeed that the energy content of the invention of a new variety should tend toward zero in the long run.

3.1.2 $\lambda(m)$ is independant of m, i.e. $\bar{\lambda} = 0$

Since $\bar{\lambda} = 0$ implies that $\lambda(m_t) = c_\lambda$ (a strictly positive constant), $\mu_t + \lambda_t \geq c_\lambda > 0$ and Point 1 of Proposition 1 makes clear that no long-run quantitative growth is then possible. The next proposition[11] follows straigthforwardly as a corollary to Proposition 1:

Proposition 2 *In an economy where the energy intensity of production activities ($\lambda_t + \mu_t$) is bounded from below by a strictly positive constant, a BGP with a strictly positive growth of final output can only rely on non-quantitative growth: it is a growth path along which Y_t is constant and where n_t, and thus F_t, grow at a strictly positive rate. Such a non-quantitative growth path is possible only if the energy intensiveness of the research process tends to zero as $n_t \to \infty$.*

The asymptotic value of Y_t it then equal to

$$Y_\infty = \frac{E_f}{\mu_\infty + c_\lambda},$$

where $\mu_\infty = \lim_{t\to+\infty} \mu(n_t) \geq 0$. Then, $F_t = n_t^\beta Y_\infty$ (see (25)), the long run growth of F_t being purely non quantitative. As recalled in Proposition 2, this non-quantitative growth path will be possible only if Point 2 of Proposition 1 is satisfied.

[11] Note that it also holds if $\bar{\lambda} < 0$ but $\mu(n_t)$ is bounded from below by a positive constant, i.e. if the energy content of an input unit remains strictly positive in the long run.

This purely non-quantitative growth path is very close to the one obtained by Grossman-Helpman (1991, section 3.2). It is also similar to the long-run growth path described in Fagnart-Germain (2011) in a model where non-quantitative growth is the result of rising product quality instead of expanding product variety as here[12].

3.2 Positive effect of complexity on energy consumption

A positive effect of complexity ($\lambda'(m_t) > 0$) implies that $\mu_t + \lambda_t$ cannot tend toward zero (with a strictly positive m_t): no quantitative long-term growth is thus possible. But may non-quantitative growth remain possible?

3.2.1 Weak complexity effect: $0 < \overline{\lambda} < \beta$

In this case, the equality $m_t = n_t$ remains but the denominator of the right-hand-side of (24) is increasing in n_t. Y_t thus follows a decreasing trend along a BGP with an increasing variety:

Proposition 3 *In an economy where the increasing complexity of final production has a positive but sufficiently weak impact on the energy intensiveness of final production (i.e. if the elasticity of $\lambda(m_t)$ is positive but smaller than the elasticity of $\upsilon(m_t)$), a BGP with a strictly positive growth of final output relies on non-quantitative growth (growth of n_t) and quantitative de-growth (negative growth of Y_t). Such a non-quantitative growth path is possible only if the energy intensiveness of the research process tends to zero as $n_t \to \infty$.*

From (25) indeed,

$$F_t = \frac{n_t^{\beta - \overline{\lambda}}}{\mu(n_t) n_t^{-\overline{\lambda}} + c_\lambda} E_f \to \frac{n_t^{\beta - \overline{\lambda}}}{c_\lambda} E_f$$

when n_t becomes sufficiently large. If $\lambda \leq \beta$, $n_t \to \infty$ implies that $F_t \to \infty$. Moreover,

$$Y_t = \frac{E_f}{\mu(n_t) + c_\lambda n_t^{\overline{\lambda}}} \to 0.$$

In very intuitive terms, $0 < \bar{\lambda} < \beta$ means that complexity brings more benefits (*via* the variety effect) than costs (*via* the increased energy intensiveness of final output). In spite of the negative growth of Y_t, the economy may thus be able to sustain a long run growth of F_t based on the variety effect (and an increasing complexity of final productions). If $\overline{\lambda} = \beta$ (variety and complexity effects of equal strenth), long run final output growth extinguishes.

3.2.2 Strong complexity effect: $\overline{\lambda} > \beta$

With functions (19)-(20) and $\beta < \bar{\lambda}$, problem (10) admits a unique interior maximum $\bar{n}_t$ given by (22). As $\mu(n_t)$ is by assumption a positive decreasing function of n_t, it is also the case of $\overline{n}_t$. As the variety used in the assembly process is $m_t = \min\{n_t, \overline{n}_t\}$, m_t is thus determined as follows:

$$m_t = \begin{cases} n_t & \text{if } n_t \leq \omega \\ \left[\frac{\beta\mu(n_t)}{c_\lambda\left[\overline{\lambda} - \beta\right]}\right]^{\frac{1}{\lambda}} & \text{if } n_t \geq \omega \end{cases} \quad \text{with} \quad \omega = \left[\frac{\beta\mu(\omega)}{c_\lambda\left[\overline{\lambda} - \beta\right]}\right]^{\frac{1}{\lambda}}. \tag{26}$$

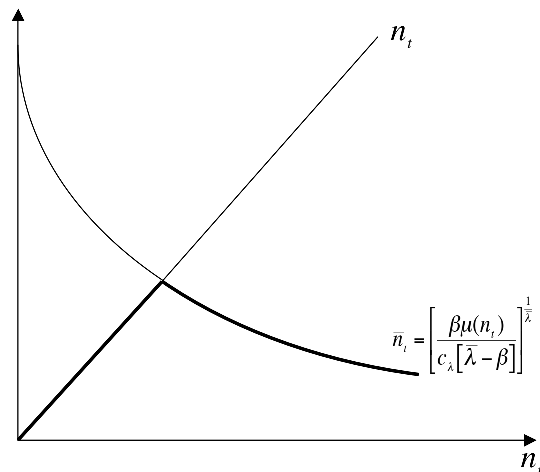
Figure 1 illustrates (26) and shows that the threshold value ω is determined by the intersection between the two curves $m_t = n_t$ and $m_t = \bar{n}_t$.

[12] One may outline a difference beween the two frameworks: in the one with expanding variety, the production of each input declines through time (a constant Y_t implies a decreasing y_t if n_t is rising). In the framework with constant variety but rising product quality, the production of each input type remains constant in the long run.

Suppose a trajectory characterised by a monotoneous increase of n_t. As long as $n_t \leq \omega$, m_t grows as n_t. But once n_t gets larger than ω, m_t decreases as n_t rises: research efforts create an increasing variety of inputs but a decreasing variety of the same inputs is used in the final production process. Such a trajectory with an increasing n_t but decreasing m_t does not satisfy our definition of a BGP and seems unreasonable anyway. There is no possible long run trajectory with rising n_t and m_t and thus no possible form of long-term balanced growth.

Proposition 4 *In an economy where the increasing complexity of final production has a positive and sufficiently strong impact on the energy intensiveness of production activities (i.e. the elasticity of $\lambda(m_t)$ is larger than the elasticity of the variety effect), neither quantitative nor non-quantitative growth is possible in the long-term. The most favourable long-term trajectory is a zero-growth path characterised by stationnary levels of variables F_t, m_t, n_t, y_t.*

Figure 1



In such a scenario, research activities vanish and the whole energy flow E is allocated to production activities. The stationary final output is thus given by

$$F = m^{\beta} \frac{E}{\mu(n) + c_{\lambda} m^{\bar{\lambda}}},$$

with $m \leq n$. In the particular case where the economy uses the maximum number of inputs in the assembly process, $n = m = \omega$ where ω is given in (26).

When $\beta < \bar{\lambda}$, an increasing complexity eventually brings more costs than benefits and a perpetual increase in m_t would be too costly. All forms of growth thus stop. This steady state solution described in Proposition 4 is similar to the long-term solutions obtained by Ayres and Miller (1980) and, more recently, by Germain (2012, section 2.1). This steady-state equilibrium could also correspond to the type of long run equilibrium favoured by the so-called degrowth literature: as Kallis et al. (2012, p.173) outline, [no]body in [this shool of thought] (...) is preaching degrowth forever: for its advocates, degrowth would be a transition path toward a lower steady-state with desirable properties (...)".

3.3 A few remarks on the purely non-quantitative BGPs

A purely non-quantitative BGP has been shown to be a possible intermediary case between the neoclassical BGP and a no-long-run-growth scenario: if the effect of complexity on energy consumption is weak (*a fortiori* if it is negative or nil), non-quantitative growth may persist (under certain conditions) in an economy where no long run quantitative growth is possible. In our framework where product differenciation is only horizontal, non-quantitative growth is exclusively driven by a variety effect. Alternatively (or complementarily), it could be the result of a rising product quality. It is thus worth outlining that conclusions very similar to those of section 3 can be obtained in a model with rising product quality *à la* Grossman-Helpman (2011, chapter 4): the same typology of growth paths and the same feasibility conditions appear *mutatis mutandis* in a model where rising product quality may be accompanied by an increased complexity of these products and/or of their production processes. But if the idea of a positive relationship between the complexity of a product (or more broadly of a system) and the variety of its components is consistent with the literature in various disciplines, we do not know any argument that establishes a systematic link between the quality of a product and its complexity. Products like cars, computers, phones,... are certainly of better quality and higher complexity today than in the past. But they are examples and not necessarily the rule.

Whatever the form of non-quantitative growth, the result of a purely non-quantitative long-term growth is fragile in three respects. We mention them here below on the basis of the model with expanding variety but we enlarge our comments to the case of a model with rising product quality. First, as already outlined, non-quantitative growth requires an unbounded energy efficiency in the research activities[13] that allow men to create new input varieties or to improve product quality. Next and more obviously, the positive variety effect (or the positive quality effect) on final output must itself be unbounded: if $v(m_t)$ was bounded from above, a BGP with a positive growth of F_t could only rely on quantitative growth and would only be possible under the condition stated in Point 1 of Proposition 1.

Finally, a non-quantitative BGP would neither be possible if there was a weak substitutability between input variety (or input quality) and input quantity in the final good production process. For example, consider the assembly technology given by

$$F_t^{\alpha} = \int_0^{m_t} \left[1 - \alpha + \frac{\alpha}{y_{it}^{\gamma}}\right]^{-\frac{1}{\gamma}} di, \qquad \text{where } \alpha \in [0,1] \text{ and } \gamma \geq 0. \tag{27}$$

Contrary to the functional defined by (1), F_t is no more homogenous of degree 1 in $y_{it}, i \in [0, m_t]$. With symmetric inputs $y_{it} = y_t, \forall i$, (27) writes as $F_t^{\alpha} = \left[[1-\alpha]\, m_t^{-\gamma} + \alpha Y_t^{-\gamma}\right]^{-\frac{1}{\gamma}}$. Final good production thus writes as a CES function of input variety m_t and input quantity Y_t (with increasing returns-to-scale)[14]. If $\gamma > 0$, i.e. if the substitutability between input variety and input quantity is weak, then a purely non-quantitative BGP is impossible: because $E \geq E_{ft} = [\mu_t + c_\lambda]\, Y_t$ and $\mu_t \geq 0$, Y_t is necessarily bounded from above, so that F_t is also bounded even if $m_t \to +\infty$. Equivalently, in a model with rising product quality, a purely qualitative BGP would be impossible if the substitution possibilities between input quality and input quantity was weak.

4 Conclusion

This note has reconsidered what type of long-term growth is possible in a model with expanding product variety *à la* Grossman-Helpman (1991) where all human activities require energy. In this framework, we have linked the complexity of final production to the number of different components (or inputs) entering into its assembly process. We have considered two cases, whether complexity is costly or not, i.e. whether product complexity increases the energy requirements of production operations or not. A balanced growth path combining "quantitative" and "non-quantitative" growth has appeared possible only if the potential of energy efficiency gains is unbounded in *all* (production and research) activities. This requires in particular a decrease (toward zero) of the

[13]This requirement is trivially satisfied if one assumes -as often in the endogenous growth literature- that research activities do not require energy.

[14]This CES reduces to the Cobb-Douglas defined by (17) (with $m_t = n_t$) when $\gamma \to 0$.

energy intensiveness of final production in spite of its increased complexity. Less optimistic assumptions unavoidably lead to less favourable long-term growth scenarii. If the energy intensiveness of intermediate and/or final productions is bounded from below by a strictly positive constant, quantitative growth is not sustainable in the long-run but a purely "non-quantitative" growth path may remain possible (i) if the impact of complexity on energy consumption is nil or not too strong and (ii) if the energy intensiveness of the innovation process (the research activities in the present model) tends toward zero. If either one of these two conditions is not met, zero-growth is the most favourable long-run scenario.

It is not obvious to assess the realism of the conditions under which long-term growth (even limited to its "non quantitative" dimension) is possible. First, even though common perception suggests an increasing complexity of human productions and processes, and of the economy as a whole, we do not have at our disposal an "objective" index of the complexity of our economies. *A fortiori*, we do not have a quantification of the link between complexity and energy intensiveness at the aggregate level. However, the present note tends to reinforce the pessimistic view of ecological economics with respect to the feasibility of long-term growth: in a finite world, even the intermediary case of a purely non-quantitative long-term growth is only feasible under rather restrictive conditions, as discussed above.

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