IMPORT QUOTAS FOSTER PRODUCT IMITATION IN VERTICALLY DIFFERENTIATED DUOPOLIES

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Abstract

Krishna [89] shows that quotas may act as facilitating devices by relaxing price competition. We extend her analysis by considering the following stage game : a domestic government chooses an import quota, then a domestic and a foreign firm choose the quality of their product before engaging a price competition. Because it relaxes price competition, the quota deeply affects quality choices : when costs for quality do not rise too quickly , both firms end up choosing the same highest available quality for a wide range of quota values. It is then shown that the optimal policy for the government consists in choosing the quota level which is just sufficient to induce product imitation. Once its effects on quality choice is taken into account, the quota may thus hurt the foreign firm and increase domestic welfare.

Keywords : International Trade, Quality, Optimal Quota, Price Competition
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1) INTRODUCTION

As is well-known from contributions in Industrial Organisation, the strategic relationships between firms are heavily dependent on the nature of the strategic variable used by these firms. It is therefore not surprising that in the strategic trade policy literature, policy recommendations are dependent on the mode of competition prevailing in international markets. This is best illustrated in the seminal contribution of Krishna [89], whose key message is the following: different trade policies, namely tariff and quotas, may exhibit some strong forms of equivalence in a monopoly setting but nevertheless have completely different implications in oligopolistic industries, depending on whether firms are Cournot or Bertrand players. In particular, in a price setting context, apparently "innocent" quotas (i.e. quotas set above the level of foreign firms' sales under free trade) have the property of relaxing price competition, thereby driving profits towards their collusive level. Such a result does not apply under Cournot competition. More generally, Krishna puts forward the idea that quotas are very peculiar policy instruments because they not only change the conditions under which competition takes places but may also affect the nature of competition itself.[1]

In light of Krishna's findings, it is surprising that so few papers have further investigated the specific effects of quotas under price competition, in particular by going backwards to previous stages of the competition process. In contrast, a great deal of effort has been made in order to address this issue under Cournot competition. In particular, the effects of quotas on product qualities has been a recurrent topic of theoretical research, Das & Donnenfeld [89] and Ries [93] offer early contributions and Herguera, Kujal & Petrakis [99] is a more recent one. Existing theoretical results suggest that under quantity setting, the impact of a quota on qualities will depend on the hierarchy that prevails among products before the quota is implemented. For instance, Das & Donnenfeld [89] show that if the foreign producer sells the high quality good initially, then the domestic firm will tend to downgrade quality in the presence of the quota, whereas the contrary prevails if the foreign firm initially sells a low quality product. Whether a similar tendency would be observed under Bertrand competition is to the best of our knowledge an open question.

On the other hand, several empirical papers (for instance Feenstra [88]) suggest that for the automobile market, the Voluntary Export Restraint on Japanese cars resulted in a general quality upgrading. Aw & Roberts [86] found evidence on quality upgrading for shoe imports sold in the US. At the same time, it is fair to recognise that empirical research dealing with quota issues has often been loose on the exact specification of the price formation process[2]. More theoretical work is thus called for on this topic.

[1] A similar idea is also used in Reitzes & Grawe [94] where it is shown that market shares quotas have very specific implications under Cournot competition. Gaudet & Salant [91] show how a quota may enhance collusive outcomes under Cournot competition in oligopolistic industries.

[2] For instance, Verboven [96] & Goldberg [95] simply assume the existence of a pure strategy equilibrium in prices, an assumption which seems questionable in view of the non-existence result put forward by Krishna [89]. On the other hand, Feenstra [88] assumes price taking behaviour.

In this paper, we provide the first detailed analysis of quality choices *under price competition* in the presence of a quota. We solve for this a three stage game. In the first stage, a domestic government chooses the level of the quota in order to maximise domestic welfare. In the second stage a domestic and a foreign firm choose qualities, before price competition takes place in the domestic market at the third stage. We characterise the unique subgame perfect equilibrium of this game.

We find that the presence of the quota drastically weakens the incentive to differentiate products at the quality choice stage because it dramatically relaxes competition at the pricing stage. When quality is not costly, the quota fosters minimal differentiation: both firms choose the best available quality and therefore end up selling homogeneous goods in equilibrium for a very large domain of values of the quota. When quality is costly, firms may or may not choose identical qualities in equilibrium; nevertheless, average quality increases and the degree of differentiation drastically decreases as compared to the equilibrium outcome under free trade.

The intuition for this result is best understood by assuming that quality is not costly. The presence of the quota prevents a price competition that is too fierce, whatever the degree of differentiation. Indeed, since the foreign firm cannot sell more than the quota, the domestic producer may guarantee for himself a residual demand and a payoff that are strictly increasing in own quality. Therefore, the domestic producer has an incentive to raise quality, irrespective of the other's choice. On the other hand, the foreign producer tends to sell its quota at the highest price, which is increasing in own quality; therefore this firm also has incentives to maintain a high quality. It then turns out that this market outcome is desirable from a domestic welfare point of view. Indeed, consumers benefit from a much larger welfare as a result of the global quality upgrading, even though prices may be higher on average. Moreover, the quota ensures that the foreign firm will be "small" relative to the market size, so that the profit diversion effect is limited. Thus, in order to maximise domestic welfare the government sets a quota that is not too restrictive but is just sufficient to ensure that product imitation will result as well as to ensure enough price competition. This result is noteworthy because it qualifies Krishna's conclusions: the quota acts as a facilitating practice at the price competition stage but once its effects on quality choices are taken into account, the foreign producer is worse off than under free trade whereas domestic welfare is larger.

It is of course not surprising that barriers to trade affect the choice of product attributes. Even the possibility that firms choose identical attributes for their products has been established in the literature. For instance, Schmitt [95] also reaches a minimal differentiation equilibrium outcome but he uses a horizontal differentiation framework. A discussion of our findings in the light of previous results is thus called for. Schmitt shows that when barriers to trade are high, each producer concentrates on its domestic market. Product imitation obtains in equilibrium because it guarantees that a foreign entrant would make no profit in the domestic market. Thus, identical products are chosen in equilibrium because they fully prevent further price competition. And indeed, there is no trade in this equilibrium outcome. In our model however, choosing homogeneous products does not amount to relaxing price competition at all and trade always occurs. It is really the nature of the trade restriction within the price competition framework which completely reverses firms' incentives with respect to quality choices. In Boccard & Wauthy [97a] we study the impact of a quota in a horizontally differentiated industry. We show how the quota relaxes price competition in this case but no tendency towards minimal differentiation is to be expected under horizontal differentiation.

As for vertical differentiation, Herguera & Lutz [98] survey leapfrogging issues under strategic trade policies. Our results point in the same direction as theirs since the effect of the quota on firms' best replies at the quality stage could be viewed as inducing leapfrogging. Das & Donnenfeld [89] and Herguera, Kujal & Petrakis [99] also deal with quality choices but under quantity competition. We offer a direct counterpart to their work under price competition and our results indeed bear some resemblance to theirs. We postpone however a detailed comparison until the end of section 4.

We model quality differentiation using the approach pioneered by Mussa & Rosen [78], Gabszewicz & Thisse [79] and Shaked & Sutton [82] i.e., the "address-model" approach to vertical differentiation. Interestingly enough, this is precisely the framework retained by most of the previous work assessing the implications of quotas on products' qualities under imperfect competition (Das & Donnenfeld [87], [89], Krishna [87], [90], Ries [93], Herguera, Kujal & Petrakis [99]). By considering a vertically differentiated duopoly, our analysis could thus be viewed as a complement of Krishna [89], which deals with horizontal differentiation. Specifically, the asymmetry that characterises vertical differentiation leads to an analysis of the pricing games that departs from Krishna [89].

One of the key feature of our analysis is that it solves the Nash equilibrium in prices for the differentiated products *over the whole range of admissible quotas*. In contrast, Krishna [89] provides a local (i.e. in the vicinity of free trade) analysis under horizontal differentiation. Lutz [97] deals with vertical differentiation but his analysis also is a very local one.[3] As will appear soon, such a local analysis in the vicinity of the free trade equilibrium level often turns to be quite misleading.

Last, it should be mentioned that our analysis is related to the literature dealing with price competition and capacity constraints. As made clear in Krishna [89] and nicely summarised by a referee "a quota is in some ways similar to a forced capacity pre-commitment". In this respect, our analysis of pricing games can be viewed as an extension of Levitan & Shubik [72] for the case of a vertically differentiated industry, i.e. we solve a class of price subgames where one of the two firms is capacity constrained while the other enjoys an arbitrarily large capacity. Note also that the no-differentiation limiting case of our model is precisely given by the linear model of Bertrand competition they analyse. Interestingly enough, price competition and capacity commitment has been extensively studied in the case of homogeneous products (see in particular Kreps & Scheinkman [83]) but the extension towards differentiated industries remains largely unexplored. Furth & Kovenock [93] and Boccard & Wauthy [97a], [97b] provide contributions in the case of horizontal differentiation but the present work is the first to deal with a vertically differentiated industry.

The paper is organised as follows. In section 2, the vertical differentiation model is introduced. The Free Trade equilibrium is stated and we discuss the specific implications of a quota under vertical differentiation. Section 3 is devoted to the analysis of price competition in the presence of a quota with a distinction according to which firm has a low quality. In section 4, we address the issue of quality choices in order to solve the full game and comment equilibrium outcomes in the light of those obtained under quantity competition. Section 5 characterises the optimal quota from a domestic point of view. Section 6 concludes. Proofs are relegated to the appendix.

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[3] We thank a referee for noticing this paper and are grateful to Stefan Lutz for sending us his unpublished manuscript.

2) THE SET-UP

Let us consider a two-stage game with quality choice in the first stage. In the second stage, two firms, **l** and **h**, sell indivisible goods differentiated by their quality indexes s_l and s_h, which satisfy $s_l < s_h$. Firms produce at zero cost and maximise profits by setting prices p_l and p_h non-cooperatively. Consumers are willing to buy at most one unit of the good and exhibit heterogeneous preferences. They are identified by their taste for quality **x** which is uniformly distributed on the interval [0;1]. The net utility of consuming good $i \in \{l,h\}$ for the consumer with taste **x** is $u_i = xs_i - p_i$ and we set the default utility of no-consumption to zero.

To address the issue of quality choices in the first stage we assume that firms choose qualities at zero cost. This assumption is made in order to isolate the pure effect of the quota on the incentive for a low quality firm to imitate a high quality one. We shall later test the robustness of this hypothesis by introducing a convex sunk cost in section 4. As a direct consequence of zero cost for quality we have to assume that the range of possible qualities is exogeneously bounded with $s_i \in [0,1]$.[4]

2.1 FREE TRADE EQUILIBRIUM

The Subgame Perfect Equilibria of the game described above define the Free Trade benchmark for the analysis to follow. Lemma 1 recalls of the nature of these equilibria.

Lemma 1

In the quality-price game, there are two subgame perfect equilibria: one firm chooses the best available quality (i.e. s_i =1) while the other one differentiates to a ratio of 4/7. Whatever the quality hierarchy, the price equilibrium of the continuation game is unique and in pure strategies.

A proof of this standard result can be derived from Choi & Shin [92]. Recalling the essence of price competition under vertical differentiation will be useful for the analysis to follow. This can be done referring to Figure 1 below. It depicts firms' best replies in a pricing game associated with quality choices $s_l < s_h = 1$. Note first that the price space must be partitioned in two: the duopoly region where both firms enjoy a positive market share and the monopoly region where only the high quality firm enjoys a positive demand. Since products exhibit different qualities, the low quality firm must offer a sufficient discount with respect to the high quality price to compensate for the quality differential. The required discount defines the frontier $p_l = p_h s_l$ between the monopoly and duopoly regions.

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[4] Letting X be the upper bound on taste x (i.e., income), the maximum willingness to pay for quality **s** is linear in **s** while the investment in R&D that enables to achieve **s** is convex. There is thus an upper bound S that even a monopoly would not select. Alternatively S can be though of as the best already patented quality that a firm can buy on a short term base. Our normalisations of M =1 and S =1 may seem at odds with most of the literature (see in general Motta [92]) but it is intended to clarify the exposition of the price game and to highlight the effect of the quota. In section 4 we test the robustness of our analysis by introducing a sunk cost for quality s^2/F. Notice that a large F (low cost) is of course similar to a large M .

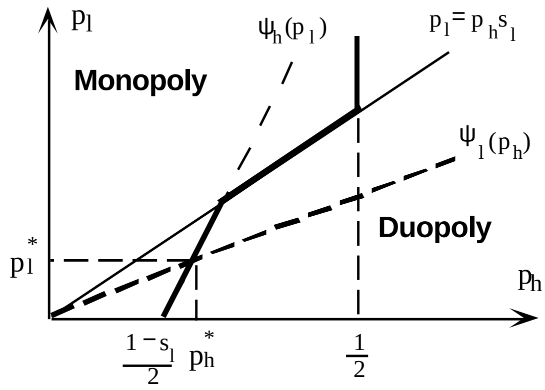


Figure 1

Obviously, the low quality firm's best reply (denoted by $\psi_l(p_h)$) lies in the interior of the duopoly region. In contrast, the best reply of the high quality firm is in the duopoly region only against low p_l. Against higher prices, the high quality firm optimally excludes the low quality product and enters into the monopoly region. Either by naming the limit price which is just sufficient for this purpose (in which case its best reply is at the frontier between the two regions) or by naming the monopoly price(which is 1/2 in the present case). Note thus that the best reply of the high quality firm exhibits three linear segments. Under free trade, only the first one is relevant so that the equilibrium of the pricing game is invariably defined in the duopoly region. As will appear in the forthcoming analysis, in the presence of the quota, strategies involving market pre-emption, i.e. defined along the second or third segment, will be part of a price equilibrium. The detailed construction of the best replies is fairly standard and can be found in appendix A.1.

2.2 Quota, Rationing and quality hierarchy

As pointed out in Krishna [89], a quota tends to destroy the existence of a pure strategy equilibrium in pricing games. Indeed, it generates spillovers to the benefit of the domestic firm which typically destroy the concavity of payoffs. The argument is easy captured. In the presence of the quota, there exist price constellations for which the quota is strictly binding. Typically, this happens if the foreign producer charges a low price against a relatively high domestic price. In that situation, some consumers are rationed by the foreign producer but can transfer their purchase on the domestic producer. It provides the domestic producer with an incentive to strategically raise its price, anticipating that some rationed consumers will be recovered. This strategy may be referred to as "hiding behind the quota". Note that its profitability (relative to the more standard aggressive strategy) depends on the propensity of rationed consumers to transfer their purchase to the domestic firm instead of refraining from consuming, i.e. on the importance of the spillovers, which in turn depend on who the rationed consumers are and on the substitutability of the goods.

The spillover argument was put forward in Edgeworth's critique addressed to Bertrand. Recall that pricing games in the presence of a quota are formally equivalent to a pricing game where one firm faces a capacity constraint. It is a standard result in this literature that equilibrium outcomes are heavily dependent on the rationing rule (see in particular Davidson & Deneckere [86]) and the present analysis is no exception. In what follows we adopt the so-called efficient rationing rule. As defined by Tirole

[88], efficient rationing assumes that rationed consumers are those who exhibit the lowest taste **x** for the product.[5] In the context of a differentiated market the rule must be somewhat re-interpreted. We assume the following: once firms have set their prices, consumers ask for the product that maximises their utility. If demand for the foreign product exceeds the quota, consumers with the lowest reservation price for the foreign products are rationed but try to achieve their "second-best" i.e., turn to the domestic producer or refrain from consuming. Under this rationing rule, the specific implications of a quota taking place under vertical differentiation are easy to trace.

Let **d** and **f** denote the domestic and foreign producers. We provide one of the main insight for the analysis by showing how the extent of the spillover is related to the hierarchy of product qualities.

Consider Figure 2 which depicts a market configuration in which $s_d < s_f$. At the prevailing prices, the demand addressed to the foreign firm exceeds the quota **q**. Observe that any consumer willing to buy the high quality product always prefers buying the low quality one to refraining from consuming. Hence, *all consumers rationed by the foreign firm will be recovered by the domestic firm whatever the rationing rule.*

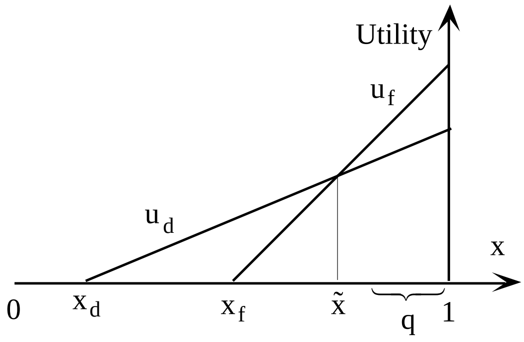


Figure 2

Consider now a market configuration with $s_d > s_f$ and the same rationing problem as depicted in Figure 3 below. Agents located on the left of x_d prefer to refrain from consuming rather than buying from firm **d**, if rationed by firm **f.** Hence, the nature of the rationing rule matters. Our present assumption induces the lowest possible recovering of the rationed consumers.

The bold segment on Figure 3 is the fraction of rationed consumers recovered by the domestic firm. It sells to consumers located in $[\tilde{x};1]$ and in $[x_d;\tilde{x}-q]$, thus its demand is $1-q-x_d$ i.e., it behaves as a monopolist serving a market of restricted size $1-q$. It is also clear that if p_d or **q** increase enough, none of the rationed consumers are recovered by the domestic firm.

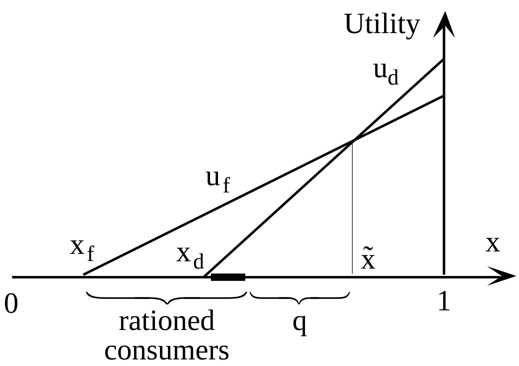


Figure 3

Two remarks are called for at this step. As opposed to the example of Figure 2, our rationing rule does not maximise consumer surplus when Figure 3 applies, so the term "efficient rationing" is not truly appropriate here. Still, if another rationing rule is used our model is unaffected as long as arbitrage is legal. More precisely, what we require is that after prices have been quoted, consumers who have been allocated the foreign product may arbitrage at no cost and turn afterwards to the domestic product. Indeed the rationed consumers with high reservation price like those in $[x_d+q;\tilde{x}]$ could buy the low

[5] This rationing rule has been used for instance by Kreps & Scheinkman [83] and easily compares with the implicit rule considered by Krishna [89].

quality good from low reservation price consumers like those in $[x_f;x_d+q]$ before turning to the high quality firm. This arbitraging process yields the same outcome for firms as our rationing rule. Yet some arbitraging possibilities remain. Indeed consumers located in $[x_d;\tilde{x}-q]$ could trade their high quality good with the low quality good of those located in $[x_d+q;\tilde{x}]$ and this would increase total consumer surplus. The crucial point for us is that market shares remain **q** and $1-q-x_d$.

Summing up, we note that in a vertical differentiation framework, the profitability of "hiding behind the quota" crucially depends on the quality hierarchy prevailing between the two products. The spillover on the domestic demand associated with the presence of rationing is more systematic when the domestic producer sells the low quality and should influence more strongly the equilibrium of the pricing game in the presence of the quota. This is independent of the particular rationing rule retained for the analysis but reflects an asymmetry that is fundamentally rooted in the vertical structure of the model. As a direct consequence, the two possible cases deserve separate analysis for the pricing games. They are both considered in section 3.

3) PRICING GAMES WITH A QUOTA

In this section, we characterise price equilibria in all possible price subgames. The full analysis of the price subgames in the presence of the quota is long and rather involved. We have tried to keep exposition as simple as possible and most of the intuition can be captured by referring to the figures displayed in the body of the text. With a high quality foreign product, we may directly apply the analysis of Krishna [89] to the case of a vertically differentiated industry (subsection 3.1). However, the analysis of the low quality foreign product case (subsection 3.2) departs from Krishna and offers more general insights into the analysis of pricing games with quantitative constraints which, to the best of our knowledge, are new. More precisely we identify two types of equilibria that were not present in Krishna [89]. The first one is a pure strategy equilibrium in which the quota is strictly binding whereas the second one involves both firms using mixed strategies.

3.1 THE CASE OF A HIGH QUALITY FOREIGN PRODUCT

We start by considering the case where the foreign product is the high quality product. In the presence of the quota, the nature of price competition is best understood referring to Figure 4.

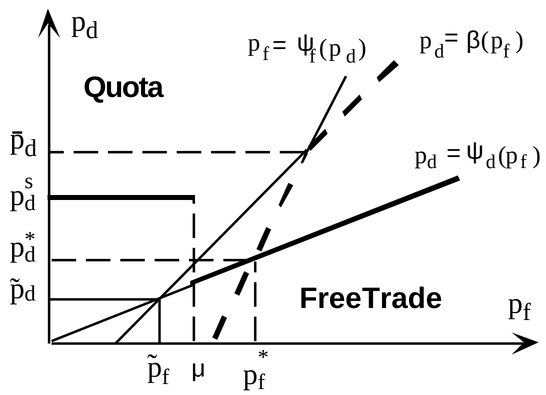


Figure 4

In order to simplify the exposition we assume without loss of generality that $s_f = 1$. Note first that the space of prices can then be divided in two regions , "Free Trade" and "Quota", according to whether, at the corresponding prices, demand addressed to the foreign firm exceeds the quota or not. If p_f is large relative to p_d, the quota is not binding. Yet, decreasing p_f relative to p_d will at some point make the quota binding. We identify by $p_d = \beta(p_f)$ the set of prices such that the quota is exactly binding. Second, when p_f is low relative to p_d, the quota is strictly binding. The low quality firm is not excluded from the market as was the case under free trade (cf. Figure 1 above). Instead, it is recovering all the rationed consumers and becomes a monopoly over a restricted market of size $1 - q$. As for the foreign firm, it faces a constant demand equal to the quota.

There are thus two possible competition regimes depending on whether the foreign price is low relative to the domestic one or the reverse prevails. In the first case, the quota is binding and rationing is at work. The best a foreign producer can do is to sell its quota at the highest possible price (i.e. naming $p_d = \beta(p_f)$), whereas the domestic producer acts as a monopolist along the residual demand. This corresponds to naming p_d^s, a strategy we will refer to as the "security" strategy. In the second case, the free trade analysis applies: firms fight for market shares. As shown by Krishna [89], the essence of the price competition lies in the fact that the domestic firm is balanced between these two alternatives against the foreign price. In Figure 4, this is materialised by a discontinuity in the domestic firm's best reply at price μ. This discontinuity reflects thus the fact that against low foreign prices, the domestic producer tends to avoid price competition by hiding behind the quota whereas against high prices, it becomes more profitable to fight for market shares by naming low prices. This discontinuity precludes the existence of a pure strategy equilibrium for many constellations of quota and product attributes. In contrast, the foreign firm's best reply is continuous.

In the present setting, we can safely apply the methodology laid out in Krishna [89]: when a pure strategy equilibrium does not exist, the only equilibrium candidate sees the domestic firm mixing over two prices, p_d^s and $\varphi(\mu)$ against the pure strategy μ for the foreign firm. In Proposition 1 we characterise the "Krishna" equilibrium for the case where the foreign product is the high quality one. Since the formal argument offers no specific novelty, the proof of Proposition 1, including the derivation of firms' best reply, has been relegated to appendix A.2. For loose quotas, the free trade equilibrium prevails whereas the equilibrium involves the domestic producer using a mixed strategy when the quota is more restrictive.

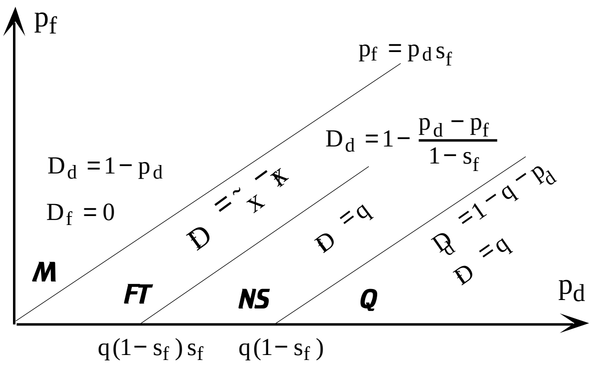
Proposition 1

Assume $s_d < s_f = 1$. The price equilibrium is unique and there exists a threshold quota $q^*(s_d)$ such that
- if $q \geq q^*(s_d)$, the free trade equilibrium prevails
- if $q < q^*(s_d)$, the domestic producer randomises over p_d^s and some lower price while the foreign producer plays the pure strategy $\mu(q,s_d)$.

3.2 THE CASE OF A LOW QUALITY FOREIGN PRODUCT

We assume now that $s_d = 1$ and $s_f < 1$.[6] Two additional problems come into play and lead to two new results. First, there always exist price constellations for which the quota is strictly binding while the domestic producer is recovering no consumer, i.e. does not benefit from any spillover. As a consequence, a quota set at the free-trade level may be ineffective whereas a quota just below it may induce a pure strategy equilibrium. Second, since the domestic firm is now the high quality firm, it benefits from the possibility of excluding the other firm of the market. Although this possibility is never used under Free Trade we will show that it becomes relevant in the presence of a quota, with the major implication that in equilibrium, both firms use mixed strategies for some configurations of quota and qualities.

As previously, we develop an informal argument relying on graphical illustrations. Then we proceed to the formal derivation of the equilibrium. Consider first Figure 5 below which identifies the possible configurations of firms' sales (see appendix A.3 for the detailed characterisation of these functions). There are four regions of interest in the price space. Think of a fixed price for the foreign firm and start increasing the domestic price from 0. Since the foreign product is the low quality one, preempting the market is an available option for the domestic producer. This occurs in the "Monopoly" region **M**. For intermediate prices, the foreign producer enjoys a positive demand, which is low enough to comply with the quota. This is the "Free Trade" region **FT**. When the quota is strictly binding, rationing prevails. Two configurations may then obtain. Either prices are such that rationed consumers do not report their purchase on the domestic firm (recall of Figure 3). This is the "No Spillover" region **NS** where the foreign producer sells the quota whereas the domestic one enjoys the free trade demand. Or spillovers accrue to the domestic firm: this is the "Quota" region **Q**. Figure 5 below depicts these four regions with the associated demand functions.



<u>Figure 5</u>

As for best replies, the intuition identified in the previous subsection is still at work: the domestic firm is balanced between pricing aggressively and retreating along the residual demand it can secure given the quota. However, two additional problems must be taken into account. First, because there exists a region where the quota is binding while no-spillover occur, we must consider as a

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[6] But the argument obviously applies to any configuration where $s_f < s_d \leq 1$.

possible candidate the case where the domestic producer fights for market shares while the foreign firm sells the quota. Second, because market pre-emption is an available option for the domestic firm, it may happen that the domestic producer wishes to randomise between its security price and the limit price. As shown below, when this occurs, the semi-mixed equilibrium identified by Krishna cannot apply.

Let us first show that the NS region always exists. The frontier between NS and FT is defined as the solution to $\tilde{x} - x_f = q$ is $p_f = \gamma(p_d) \equiv p_d s_f - q s_f (1 - s_f)$; if p_f becomes lower than this benchmark, the quota becomes binding and under efficient rationing, the rationed consumers are located in the interval $\left[x_f ; \tilde{x} - q\right]$. The domestic firm will benefit from spillover if and only if

$$x_d \le \tilde{x} - q \;\Leftrightarrow\; p_f \le p_d s_f - q(1 - s_f)$$

As soon as the foreign firm sells the lower quality product, the region "NS" depicted in Figure 5 above is non void: the domestic firm does not recover any of the rationed consumers, thus its demand function is still the free trade one, whereas the foreign producer is already constrained and thus sells q. The demands in regions "M" and "FT" have been characterised in section 2 and appendix A.1 . For region "Q", the domestic producer benefits from spillovers and has a monopolistic demand over the market share $1 - q$ as previously explained in sub-section 3.1.

We now turn to the characterisation of the best replies. The case of the foreign producer is easy. The demand being nil in region "M", anything is a best reply but is also dominated by the best reply of the "FT" region. Foreign demand is equal to **q** in region "NS" as well as in region "Q", thus the best reply is the frontier price $\gamma(p_d)$ which is itself dominated by the candidate of the "FT" region. The foreign best reply is thus the free trade best reply $\psi_f(p_d) = \frac{p_d s_f}{2}$ when it is interior and the frontier price $\gamma(p_d)$ otherwise ; it is continuous with a kink at $\bar{p}_d \equiv 2q(1 - s_f)$, the intersection between $\psi_f(.)$ and $\gamma(.)$. Formally :

$$\varphi_f(p_d) = \begin{cases} \psi_f(p_d) & \text{if } p_d \le \bar{p}_d \\ \gamma(p_d) & \text{if } p_d > \bar{p}_d \end{cases} \tag{1}$$

The analysis of the domestic firm's best reply is more involved. In region "Q", her profit reaches a maximum of $\frac{(1-q)^2}{4}$ for the "security" price $p_d^s \equiv \frac{1-q}{2}$. Throughout regions "NS", "FT" and "M", the demand takes the form already seen in section 2, thus the *candidate* best reply is given by equation (6) :

$$\varphi_d(p_f) = \begin{cases} \frac{p_f + 1 - s_f}{2} & \text{if} \quad p_f \le \frac{s_f(1-s_f)}{2 - s_f} \\ \frac{p_f}{s_f} & \text{if} \quad \frac{s_f(1-s_f)}{2-s_f} \le p_f \le \frac{s_f}{2} \\ \frac{1}{2} & \text{if} \quad \frac{s_f}{2} \le p_f \end{cases} \tag{2}$$

As in the previous subsection, there are two kind of strategy profiles : "hiding behind the quota" with p_d^s and fighting for market shares with $\varphi_d(p_f)$ (even though it may involve pure monopoly pricing). Recall however that the latter best reply is kinked. Therefore, to identify the benchmark price p_f which makes the domestic firm indifferent, we must however take into account that $\varphi_d(p_f)$ may be interior to region "FT" or lie on the frontier between "M" and "FT". Thus we have to analyse two equations:[7]

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[7] There is no third case because pure monopoly pricing cannot be dominated by monopoly pricing over a restricted market.

$$-\frac{(1-q)^2}{4} = \Pi_d\left(\frac{p_f+1-s_f}{2}, p_f\right) = \frac{(1-s_f+p_f)^2}{4(1-s_f)} \qquad p_f = \rho(q,s_f) \equiv \sqrt{1-s_f}\left(1-q-\sqrt{1-s_f}\right)$$

$$-\frac{(1-q)^2}{4} = \Pi_d\left(\frac{p_f}{s_f}, p_f\right) = \frac{p_f}{s_f}\left(1-\frac{p_f}{s_f}\right) \qquad p_f = \lambda(q,s_f) \equiv s_f\,\frac{1-\sqrt{q(2-q)}}{2}$$

The benchmarks are relevant if the second is greater than $\frac{s_f(1-s_f)}{2-s_f}$ itself greater than the first. Those inequalities lead to $q < \hat{q}(s_f) \equiv 1 - 2\frac{\sqrt{1-s_f}}{2-s_f}$ and $q > \hat{q}(s_f)$ so that there is always one and only one of them applying for every possible combination of **q** and s_f. The best reply function of the domestic firm will thus take one of the two following form depending on the level of the quota and the degree of product differentiation:

$$q < \hat{q}(s_f) \qquad \varphi_d^{\lambda}(p_f) = \begin{cases} p_d^s & \text{if} \quad p_f \le \lambda(q,s_f) \\ \frac{p_f}{s_f} & \text{if} \quad \lambda(q,s_f) \le p_f \le \frac{s_f}{2} \\ \frac{1}{2} & \text{if} \quad \frac{s_f}{2} \le p_f \end{cases} \tag{3}$$

$$q \ge \hat{q}(s_f) \qquad \varphi_d^{\rho}(p_f) = \begin{cases} p_d^s & \text{if} \quad p_f \le \frac{s_f(1-s_f)}{2} \\ \psi_d(p_f) & \text{if} \quad \frac{s_f(1-s_f)}{2} \le p_f \le \rho(q,s_f) \\ \frac{p_f}{s_f} & \text{if} \quad \rho(q,s_f) \le p_f \le \frac{s_f}{2} \\ \frac{1}{2} & \text{if} \quad \frac{s_f}{2} \le p_f \end{cases} \tag{4}$$

Expressions (3) and (4) are illustrated by Figures 6a-6b below where the foreign best reply function is displayed in bold and dashed while the domestic one is in bold and plain.

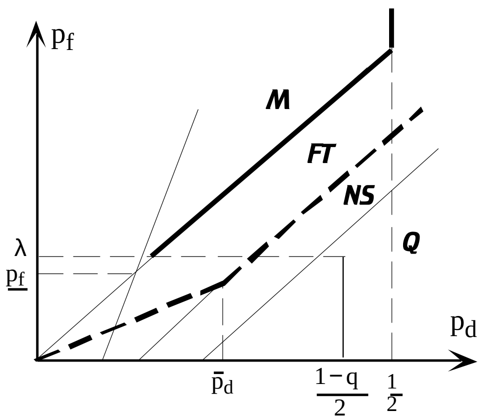

Figure 6a

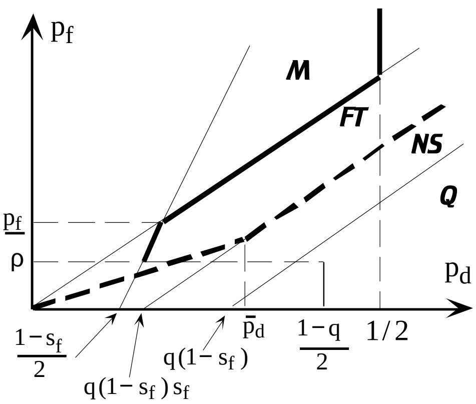

Figure 6b

With these best replies in hand, we are now able to characterise the pricing equilibrium in the following proposition which is illustrated in Figure 7.

Proposition 2

Assume $s_f < s_d = 1$. An equilibrium exists and its features depend on the combination of the quota value and the degree of product differentiation:

A) *the free trade equilibrium prevails for large quotas, which are thus ineffective*
B) *a pure strategy equilibrium exhibiting a strictly binding quota prevails for smaller quotas and large quality differentials,*
C) *a semi-mixed strategies equilibrium prevails for more restrictive quotas and intermediate quality differentials,*
D) *completely mixed strategies prevail for small quotas and small quality differentials.*

The proof of proposition 2 along with a characterisation of the nature of all possible equilibria has been relegated to appendix A.3. Figure 7 displays the four types of equilibria in the (q,s_f) space, under the normalisation $s_d = 1$. Proposition 2 is a counterpart to Proposition 1. Their comparison shows that in order to assess the impact of a quota in a pricing game of vertical differentiation *the quality ranking does matter.* In particular, we observe two new kind of equilibria, one in area B and the other in area D. Since they correspond to two previously uncovered equilibria, we provide some intuitions as to why they occur.

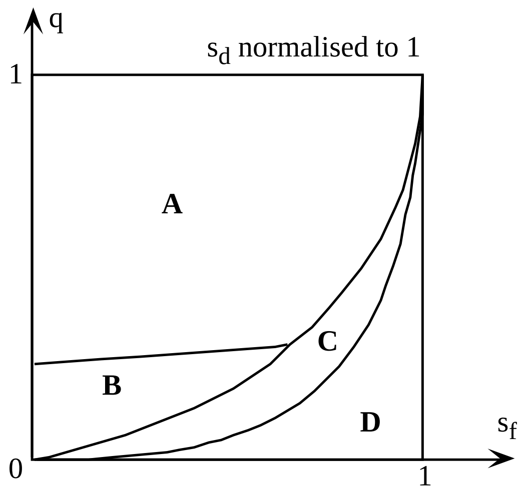

Figure 7

Consider first the frontier between A and B. It is defined by the level of demand addressed to the foreign firm under free trade. Dealing with quota levels in the vicinity of free trade amounts to analysing a region slightly above or below the A-B frontier. From proposition 2 the conclusion is immediate: whenever the degree of product differentiation is large enough ($s_f < s_d$) a quota at or just above the free trade is completely ineffective whereas a quota just below it leads to a pure strategy equilibrium. It is easy to check however that both firms' prices and profits are above the Free Trade ones. Region B displays a very clear example of Krishna's theorem 5: because there are no associated spillovers the quota does not change the nature of price competition. The foreign firm's best reply exhibits a kink when the quota becomes binding but the domestic one is unaffected. Thus, if set above free trade the quota is ineffective whereas if set below free trade it induces the foreign firm to sell the quota at the highest price which in turn allows the domestic firm to sustain a higher price in equilibrium. This no-spillover effect is obviously dependent on the rationing rule we have retained since it comes mainly from the fact that rationed consumers are located at the lower end of the reservation price distribution. It illustrates however the more general point according to which rationing spillovers are significantly larger when the foreign firm sells the high quality product. This reflects the fundamental asymmetry that characterises firms under vertical differentiation.

Let us then consider region D. From a methodological point of view, the existence of this region tells us that mixed-strategy pricing equilibria as identified by Krishna do not always exist, in particular when quotas are not restricted to the vicinity of Free Trade. The reason why this equilibrium fails to exist is easy to capture. The existence of the semi-mixed equilibria relies on the fact that the foreign firm payoff against a mixed strategy of the domestic one remains concave in own price (see Krishna [89] proof of theorem 2). This argument breaks down in the present setting. When the domestic firm mixes over the security price and the limit price (Figure 6a is relevant) the foreign firm payoffs cannot be locally concave at its pure strategy candidate best reply $\gamma(p_d)$. Indeed the payoff is defined as a linear combination of a strictly increasing function ($\gamma(p_d)Q$) and zero ($\gamma(p_d)0$), i.e. it is strictly increasing. Still payoffs being continuous, there must exist an equilibrium, which must be completely mixed since it can be neither pure nor semi-mixed. Interestingly enough, the nature of the problem involved here is not specific to our vertical differentiation. What matters is the fact that because of the quota, a limit pricing strategy becomes attractive for the domestic firm. In a vertical differentiation framework, this occurs systematically because of the dominance of the high quality firm but a similar argument can be shown to apply under horizontal differentiation.[8] However, what is specific to our vertical differentiation set up is that completely mixed strategies equilibria exist only if the foreign firm sells the low quality product. Indeed, only the high quality product can exclude the other one. Again, this fact illustrates the importance of asymmetries induced by vertical differentiation.

4) PRODUCT SELECTION

We expect the presence of barriers to trade to affects firms' incentives at the quality stage since we have just shown that they fundamentally alter payoffs in the second stage of the game. We show in this section that the quota has a drastic, and to a large extent unexpected, consequence on the selection of equilibrium attributes. Essentially, *the presence of the quota fosters minimal differentiation.*

The analysis of the pricing games revealed that the structure of firms' payoffs in the second stage was heavily dependent on the quality hierarchy prevailing between the domestic and the foreign products. Therefore, in order to identify a subgame perfect equilibrium we first have to study firms' incentives under both hierarchies, i.e. we study each firm's best replies under the assumption that the foreign product is of higher quality than the domestic one (Lemma 2) and then under the alternative quality hierarchy (Lemma 3). With the help of the two corresponding lemmas, we characterise the unique subgame perfect equilibrium of the game.

Let us start with the analysis of firms' incentives in configurations where the foreign firm sells the high quality product. Since qualities matters in the demand functions only through the ratio $\frac{s_d}{s_f}$, an increase of s_d under the normalisation $s_f = 1$ is either a real increase of s_d or a decrease of s_f. Proposition 1 has solved the pricing game when the high quality product is the foreign one. It then remains to study how the resulting profits vary over the domain $\{s_f = 1, s_d < 1\}$.

[8] We develop more systematically this point in Boccard & Wauthy [97b] within the Hotelling model. In the present paper we do not need to characterise the completely mixed strategies equilibria. Note however that they differ markedly from those prevailing when products are homogeneous. In the latter case indeed, mixed strategy equilibria often takes the form of densities whereas under product differentiation firms only uses atoms (cf. Boccard & Wauthy [97b]).

Lemma 2

Over the domain {$s_f = 1$, $s_d < 1$}, the domestic firm imitates the foreign one with $s_d = s_f$ for low quotas, whereas for quota larger than $\bar{q}$ = 71% of the market size, it differentiates with s_d = 4/7. The foreign firm wishes to maximise its quality level.

The detailed proof of Lemma 2 has been relegated to appendix A.4. However its basic intuition is easy to capture. As shown in Proposition 1 there are two types of equilibria in the relevant pricing game. Against the high foreign quality, the domestic quality choice will select the kind of equilibrium for the last stage of the game. Whenever the pure strategy equilibrium applies, the domestic incentives are equivalent to the Free Trade ones whereas if the semi-mixed strategy equilibrium applies, we can show that the domestic payoffs is equal to its security payoffs and is monotonically increasing in quality. The domestic firm is balanced between two options: imitating the foreign producer and hiding behind the quota or differentiating and fighting for market shares. Unsurprisingly, its chooses the first option whenever the quota is restrictive since this guarantees a large enough residual demand. As for the foreign producer, it is a matter of computation to show that in case of a semi-mixed strategy equilibrium, the profit is also increasing in quality.

We turn now to the best reply of the foreign producer against a larger domestic quality s_d using the convention s_d = 1. As shown in Proposition 2 there are four possible kinds of equilibria in the corresponding price subgames. We have not characterised the completely mixed strategies equilibria so that we do not have explicit formulas for the corresponding payoffs. However, as shown in the appendix, the foreign firms' payoff in such cases is bounded above by the payoff in the homogeneous product configuration. We are then able to establish Lemma 3, proved in appendix A.5.

Lemma 3

Over the domain {$s_d = 1$, $s_f \leq 1$}, for low quotas, the foreign firm imitates the domestic one with $s_f = s_d$, whereas for quota larger than $\underline{q}$ = 65% of the market size, it differentiates with s_f = 4/7.

We can now go backward into the game tree and look at the equilibrium[9] of the game where firms simultaneously choose their quality levels before engaging a price competition in the resulting differentiated market. We obtain the following striking result of non-differentiation for any quota set under 65% of the market size. The proof combining Lemma 2 and 3 is in appendix A.6.

[9] Having no explicit formula for the first period payoffs, we are not able to prove quasi-concavity and we shall therefore look for a pure strategy equilibrium which happens to be unique.

Proposition 3

- For quotas less than $\underline{q}$ = 65% of the market size, the unique subgame perfect equilibrium of the quality choice game is Top Quality for both firms.

- For quotas between $\underline{q}$ = 65% and $\bar{q}$ = 71%, the domestic firm chooses Top Quality while the Foreign one differentiates optimally at $s_f = 4/7$.

- For quotas larger than $\bar{q}$ =71%, both $\left(\frac{4}{7},1\right)$ and $\left(1,\frac{4}{7}\right)$ are equilibria of the quality game.

The impact of the quota on firms' quality selection is best illustrated by comparing a firms' best reply under Free Trade and in the presence of the quota. The comparison is performed by relying on Figure 8. Consider the domestic firm best reply against s_f. There are two candidates: replying with a lower quality (i.e. in the upper triangle) in which case the best reply is $4/7s_f$ or with a higher quality, in which case top quality is the best candidate. Then, it remains to compare payoffs under the two regimes.

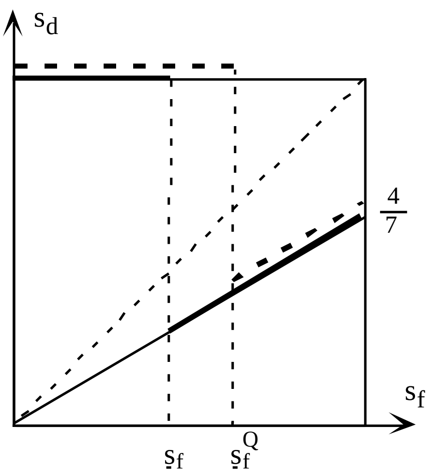

Figure 8

Computations show that there exists a critical quality $\underset{\sim}{s}_f$ below which the domestic firm is better off replying with the top quality whereas for higher foreign quality, the domestic firm accommodates with a lower quality.

This is summarised by the discontinuous best reply in bold. Assume now that a quota is implemented before firms choose qualities. How does this affect the domestic firm best reply at the quality stage? Let us consider the domestic best reply against $\underset{\sim}{s}_f$. Under free trade, the domestic firm was indifferent between replying with 1 or $4/7\underset{\sim}{s}_f$. Assume now that the quota is exactly binding in the pricing equilibrium induced by quality choices ($4/7\underset{\sim}{s}_f, \underset{\sim}{s}_f$). Relying on proposition 1, the corresponding price equilibrium is the semi-mixed one. In this equilibrium, we know that the domestic payoff is strictly increasing in s_d therefore the unique best reply candidate against $\underset{\sim}{s}_f$ must be 1. Generalising the argument, we conclude that in the presence of the quota, choosing the top quality is part of a best reply for a larger domain of s_f. In other words, the critical s_f moves to the right as depicted on Figure 8 by the discontinuous bold dashed segment. In Lemma 2 we have shown that for quotas below 71% the best reply was invariably 1 whereas for smaller values a cut-off $\underset{\sim}{s}_f^Q$ <1 exists (as depicted on Figure 8). Lemma 3 shows that a similar argument applies to the foreign quality choice with the only difference that the quota level above which accommodating in quality reappears as a valid best reply is lower.

Several comments are in order at this step. As we assumed a costless quality, equilibrium product differentiation is entirely due to price competition.[10] Under quantity competition, quality differentiation is an equilibrium outcome only to the extent that adopting a lower quality allows a firm

[10] When recasting our model in order to allow for quantity competition in the second stage, the unique equilibrium sees both firms selecting the top quality.

to save on (sunk) costs. Thus, in order to focus on the way quotas affect price competition and thereby quality choices, the zero cost for quality assumption is quite useful: it ensures that equilibrium *differentiation*, if any, *results exclusively from the presence of price competition*. We concentrate on this issue before generalising our results to convex cost for quality.

The "no differentiation" outcome reported in Proposition 3 is striking. We know of no other result which, in a purely non-cooperative setting of price competition within a single market, concludes to no differentiation.[11] This result can be explained as follows.

The first key ingredient is the mechanism that underlies Lemma 2. In the presence of a quota, the domestic firm has less to fear from price competition. Indeed, it can always hide behind the quota by using the security strategy. In a mixed strategy equilibrium, its payoffs do not depend on the action of the foreign producer but only on the level of the residual demand, which is increasing with quality. In other words, the fierce price competition that will result from product imitation is not too costly to the domestic firm. The main virtue of the quota from the domestic producer viewpoint is thus to relax price competition very drastically. So drastically in fact that its incentives to differentiate disappear. The profitability of such a strategy is only tempered by the level of the quota. Indeed, the larger the quota, the lower the residual demand associated with the security strategy. Thus for very large values of the quota, the domestic producer is better off differentiating in the standard way.

The fact that the foreign producer wishes to maximise quality seems intuitive: one expects that it is interested to sell its quota at the highest price, and thus exhibits the highest quality. Such an argument is fully compelling under perfect competition or Cournot competition. In the present context however, things are a bit more complex. Indeed, by choosing the quality level the firm chooses whether or not it will make the quota binding in the price equilibrium. When this is the case, imitating the domestic product is not strictly equivalent to maximising the quota value because this strategy induces a Bertrand-Edgeworth competition where price undercutting takes place. Prices are "fluctuating" with a range depending on the quota and this does not amount to selling the quota at the highest price. However once the domestic firm hides behind the quota in the pricing game, the foreign one benefits from a price umbrella under which it is possible to sell the quota at relatively high prices. On the other hand, when the quota becomes large, the level of prices in a mixed strategy equilibrium becomes lower and the foreign producer then prefers to differentiate with a low quality, which in this case sustains a higher (pure strategy) price equilibrium. Thus the quota offers an alternative mechanism to relax price competition. As a direct consequence, firms are less inclined to accommodate in quality. At a more formal level, it appears that *the fundamental effect of the quota is to turn Bertrand competition into a Cournot one*: the domestic firm acts as a monopolist along a residual demand parametrised by foreign demand, i.e. by the quota, exactly as it does against the other's quantity under Cournot competition.

Proposition 3 is extreme in that it concludes to product *imitation* for many values of the quota. It is fair enough to recognise that, although relying on the presence of the quota, the no-differentiation result owes much to our "zero cost for quality" assumption. If the quota allows price competing firms to sustain Cournotian outcomes, it is likely that they will differentiate if quality is costly (as they do under

[11] Friedman & Thisse [93] obtain minimal differentiation in a horizontal differentiation framework but rely on partial price collusion. Schmitt [95] reports a minimal differentiation outcome but requires two distinct markets.

a standard Cournot competition). Thus, it is important to assess the robustness of our results with respect to the assumption of zero costs for quality. To this end we assume there exist F > 0 such that a firm has to incur a cost $C_F(s) \equiv \frac{s^2}{F}$ in order to produce the quality level **s**. We maintain the assumption of an upper bound for quality at s = 1.[12] The cost is sunk when price competition takes place. We establish in appendix A.7 the following proposition that is illustrated by Figure 9.

Proposition 4

- if F > 32, both firms choose a high quality for q < $\underline{q}$ (F), the foreign firm differentiates for intermediate quota and two asymmetric equilibria with differentiation exist when q > $\bar{q}$ (F).
- if F < 32, there is no quota that induces quality imitation.

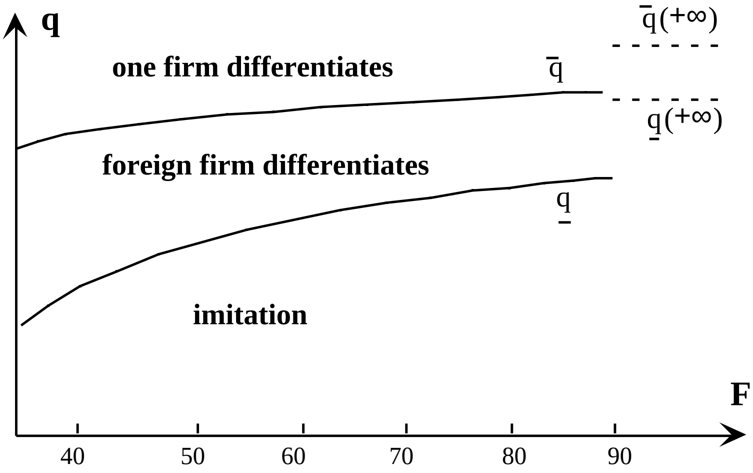

Figure 9

When quality is costly, our imitation result is thus viable provided that the cost does not rise too steeply at low quality levels (i.e., C(s) has to be "convex enough"). However the qualitative nature of our results is preserved: the degree of product differentiation sharply decreases in the presence of the quota because price competition is less intense. Moreover average quality tends to be higher than under free trade. Lastly, when the cost of quality does not allow for product imitation, it is always possible, by choosing the quota, to select the equilibrium that sees the domestic selling the high quality.

It is tempting at this step to compare our results with those obtained in previous papers dealing with standard Cournot competition. In particular, Das & Donnenfeld [89] show that the effects of the quota depends on the location of firms' respective position in the quality spectrum. Although our results differ from theirs because they consider a game where quality and quantity are chosen simultaneously, they also conclude to an increase in the quality of imports.

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[12] This standard formulation satisfies $C_F(0) = 0$ and $C_F'(0) = 0$ but since $C_F(1) < +\infty$, top quality is chosen in equilibrium for F > 60. We have therefore perform all our computations with an alternative cost function $C_F(s) = \frac{s}{F(1-s)}$ to ensure that our qualitative results still hold in the more realistic case where the top quality is infinitely costly to achieve.

Herguera, Kujal & Petrakis [99] (hereafter HKP) consider a timing similar to ours: given some quota level, quality is committed at a sunk cost before quantity competition takes place. Their results suggest that in the presence of a quota at or above free trade both firms are likely to downgrade their quality, irrespective of their position in the quality spectrum. This seem thus at odds with our findings since we roughly conclude to quality upgrading in the presence of a quota. In fact, it is not easy to compare our results with theirs' for several reasons. First, HKP [99] often distinguish two cases depending on the positions of the firms in the quality spectrum whereas in our model, products hierarchy, if any, is fully endogenous. Next HKP [99] compare two types of quantity competition whereas we compare a model of (unrestricted) price competition and one of price competition under a quantitative constraint. Recall then that the main insight of our analysis is precisely to show that from the point of view of firms' incentives towards quality choices, the main effect of the quota is to re-direct firms' choices towards those prevailing under quantity competition. The fact that this leads to less product differentiation is therefore not too surprising since it is well-known that quantity competition induces less differentiation than price competition. Quality upgrading in turn results mainly from the rise of the (possibly former) low quality firm. In HKP [99], qualities downgrade because the quota weakens the incentive to increase quality in order to steal market shares so that firms are mainly concerned by saving costs. All in all, what distinguishes our results from those obtained under Cournot competition is the following: under price competition, the impact of a quota on quality choices reflects the fact that the nature of competition in the last stage of the game has been entirely modified whereas only the intensity of it is altered under quantity competition.

The mechanism at work in the quality selection game is also reminiscent of the leapfrogging-oriented policy debate[13] (see Herguera & Lutz [98]). To a certain extent indeed, the effect of the quota is to induce leapfrogging through a reversal of a firm's best reply against a high quality. Contrarily to what happens under free trade, replying against a high quality with a lower one is no more part of a firm's best reply for many quota values. Focusing on equilibrium outcomes (instead of best replies shifts) we note that when $q < 65\%$, any firm who has a quality disadvantage has an incentive to leapfrog to top quality. However it is immediately imitated by the other firm. When $65\% < q < 71\%$, only the domestic firm will systematically leapfrog while the foreigner while never go beyond 4/7. In this last case, equilibrium leapfrogging occurs in a well-defined sense: the policy literally selects the equilibrium exhibiting the high-quality domestic product from the two possible free trade equilibria.

5) OPTIMAL QUOTA

In the preceding sections, we have shown that the presence of the quota alters both the nature of the strategic interaction in the pricing games and the nature of firms' incentives with respect to quality choices. In this section we address the issue from a normative point of view by considering the choice of the quota level by a government aiming at maximising domestic welfare. To this end, we assume that the government selects a quota level before the quality selection stage takes places.[14] It is

[13] We are grateful to a referee for attracting our attention on this issue.

[14] Note that this is game structure essentially amounts to assume that firms can adapt their qualities at no cost after the quota has been implemented by the government.

interesting at this step to recall that Krishna [89] suggests that a quota drives firms' profits towards collusive ones. Being central in establishing the possibility of *Voluntary Export Restraints*, the argument also suggests that a quota could be welfare decreasing in pricing games: it could indeed result into less production and higher prices, thereby leading to a decrease in consumers' welfare and at the same time to a larger profit diversion effect. Such a mechanism is clearly at work in the present model. However we go a step further by showing how the specification of an optimal policy must also take into account its effect on the selection of products' attributes. Our main result is stated in the following proposition, whose proof can be found in appendix A.8.

Proposition 5

Assume $F > 32$ (small cost for quality), then the optimal quota for the domestic government is $\underline{q}(F)$ while the optimal "voluntary" quota for the foreign firm is anything larger than $\bar{q}(F)$.

There are two parts in the proposition, the first one tells us that in order to maximise domestic welfare, the government should choose the largest quota which is compatible with the no-differentiation outcome. The second part tells us that the foreign producer would not choose a binding VER. The second result seems to contradict the original finding of Krishna according to which VER would act as a facilitating device to the benefits of both firms. Indeed, according to Proposition 5, a foreign firm would not commit to a Voluntary Export Restraint. It is therefore important to stress why such a contradiction with Krishna's findings obtains. When qualities are given it is of course true that quantitative restrictions increase both firms' profits and in the present setting it is direct to show that this translates into a lower domestic welfare. However, once it is recognised that these restrictions may affect firms' incentives in previous stages of the game, this negative effect does not hold anymore. In our model, the foreign producer looses much from the imposition of a quota because this quota induces a more aggressive behaviour of the domestic producer *at the product selection stage*. The presence of the quota secures positive profits for both firms even with homogeneous products (which is the basic intuition of Krishna) but the foreign producer prefers to face no quota in order to induce product differentiation at the price competition stage. In this case indeed, there is a chance that it will be the unique high quality producer (recall indeed that the Free Trade game exhibits two equilibria).

How can we interpret the nature of the optimal government policy? Our simple mechanism is developed in the limiting case of a costless quality in order to highlight intuition. The optimal quota $\underline{q}$ = 65% means that the domestic government should choose the loosest quota that is compatible with quality imitation. Notice then that the First-Best benchmark is the pure Bertrand outcome with identical products. In this case indeed, no consumers refrain from buying, all consumers buy the best available quality and the foreign firm captures no rent. In terms of aggregate welfare, both firms choosing the best available quality is obviously desirable since any degree of differentiation would lead to a lower welfare in equilibrium. Now, we know from the previous section that it is always possible to generate the no-differentiation outcome with a quota. At the same time, the corresponding price competition will not generate a pure Bertrand outcome. However, setting the largest quota compatible with the no-differentiation outcome maximises consumers welfare and minimise foreign profits. This is subject to the constraint that the foreign firm has sufficient incentives for product imitation. Setting a very restrictive quota (at the limit, a fully restrictive one) would of course foster minimal differentiation but would also restrict price competition too drastically and thus lead to a too low consumer welfare. On the other hand, setting a very loose quota (at the limit, no quota at all) would induce a product differentiation with consumer surplus losses that offset the advantage of a non regulated price competition.

It is interesting to note that in the present setting, the quota might also be viewed as a policy that selects the most desirable equilibrium. It is easy to show here that the domestic government can use the quota to force domestic leapfrogging which happens to be optimal whenever quality imitation cannot be sustained because of costs. Recall indeed that under Free Trade, there are two subgame perfect equilibria which essentially differ by the identity of the high quality producer. It is clear that from a domestic welfare point of view, the equilibrium exhibiting the domestic producer as the high quality seller is preferred. We note then from Proposition 3 that it is always possible for the government to select this outcome by choosing a quota between 65% and 71%.

6) Final Remarks

Krishna [89] made clear that under price competition, import quotas have specific implications mainly because they affect directly the nature of the strategic interaction. In the present paper we have extended this result. Using a stylised vertical differentiation framework, we have shown how the impact of the quota at the last stage of the game deeply alters firm's decisions in previous stages. With respect to quality choices, incentives are basically reversed since minimal differentiation obtains in equilibrium when quality is not costly. More generally, when quality is costly, the degree of differentiation drastically decreases in the presence of a quota. Moreover, we have shown that a domestic government benefits from enforcing such an outcome by an appropriate choice of the quota level.

It is however fair to say that these results have been derived using very specific assumptions. As a direct consequence we are not inclined to take our precise policy recommendation literally. However, we shed a complementary light on the assessment of policy restrictions put forward by Krishna [89]. The key element for it is the impact of the policy on the strategic interaction (what Krishna calls the "I" effect). What is crucial is thus to assess the impact of the policy on the strategic interaction. We go a step further by showing that even more crucial is the way policy instruments generate spillovers from the last stage of the game towards previous stages. What is puzzling in this respect is that a policy which is likely to be welfare decreasing in the last stage becomes highly desirable once its full effect over the whole sequence of decisions are studied. We have shown that it is a restrictive quota which is optimal because it induces firms to choose homogeneous goods exhibiting the highest available quality.

Other policy instruments have been studied recently in the literature on quality incentives under price competition, namely tariffs and minimum quality standards. For instance, Cremer & Thisse [94] showed in a setting quite similar to ours that taxation may have unexpected consequences on quality choices under price competition.[15] It is obviously possible to increase welfare with tariffs though,

[15] Although their model is not a strategic trade policy model, it provides a convenient benchmark since a tariff on the foreign product only would be a particular case of their analysis.

referring to their analysis, it is very likely that the optimal level of the tariff will be highly dependent on the hierarchy prevailing between products. Ronnen [91] shows that minimum quality standards may also improve welfare. However, it is clear that none of the effects underlined in our paper are present in these analysis because of the fundamental difference that exists between quotas and tariffs or minimum quality standards under price competition: *tariffs affect the magnitude of firms' incentives whereas quotas tend to affect their direction.* More generally our work confirms Krishna's insight according to which quotas tend to have very specific implications under price competition.

Let us conclude by considering the links that exists between our analysis and the literature on price competition and capacity constraints. It is well-known since Kreps & Scheinkman [83] and more recently Boccard & Wauthy [2000] that a close relationship can be established between Bertrand and Cournot models when capacity constraints are present. Recall then that a pricing game in the presence of a quota is formally equivalent to a particular form of a capacity constrained pricing game. In this respect our paper can be viewed as a preliminary step towards generalising the approach of Kreps & Scheinkman in vertically differentiated industries. The results reported in section 3 and 4 clearly suggest that the Cournotian flavour obtained with homogeneous products is likely to generalise to differentiated industries. This is largely confirmed for the case of horizontal differentiation in Boccard and Wauthy [97a]. Regarding a stage game where quality and capacity are committed at no cost before price competition takes place in a way similar to Kreps & Scheinkman, we conjecture that choosing the top quality and committing to the corresponding Cournot quantities is part of a subgame perfect equilibrium for the two firms.

APPENDIX

A.1 Price Best replies under free trade

Observe first from the set-up of the model that for $i \in \{l,h\}$, the consumer located at $x_i \equiv \frac{p_i}{s_i}$ enjoys zero utility, hence every consumer with taste $x > x_i$ is willing to buy product **i** at the price p_i. Potential markets are respectively $[x_l;1]$ and $[x_h;1]$. As a second step we identify the marginal consumer $\tilde{x}$ who is indifferent between the two products **h** and **l**. Solving for $\tilde{x}s_l - p_l = \tilde{x}s_h - p_h$, we obtain $\tilde{x}(p_l,p_h) = \frac{p_h - p_l}{s_h - s_l}$. Obviously, any consumer $x > \tilde{x}$ prefers **h** to **l** whereas the contrary prevails for $x < \tilde{x}$. Observing that quality levels can be re-scaled, we set $s_h = 1$ without loss of generality so that the demands are :

$$D_l(p_l,p_h) = \begin{cases} \tilde{x} - x_l & \text{if } p_l \le p_h s_l \\ 0 & \text{if } p_l > p_h s_l \end{cases} \tag{A1}$$

$$D_h(p_l,p_h) = \begin{cases} 1 - \tilde{x} & \text{if } p_l \le p_h s_l \\ 1 - x_h & \text{if } p_l > p_h s_l \end{cases} \tag{A2}$$

The particular shape of demands reflects the fact that in vertically differentiated markets the high quality firm may exclude the low quality one from the market. The latter, in order to enjoy a positive market share, must quote a price p_l significantly lower than p_h to compensate for its lower quality ($p_l < p_h s_l$). Note also that since $x \in [0,1]$, the market cannot be covered in equilibrium, expect perhaps for the case where $s_l = 0$.[16] Using equations (A1) and (A2), best replies are easily derived.

The profit function of the low quality firm **l** is

$$\Pi_l(p_h, p_l) = p_l D_l(p_h, p_l) = p_l \frac{p_h s_l - p_l}{s_l(1-s_l)} \tag{A3}$$

The solution to $\frac{\partial \Pi_l}{\partial p_l} = 0$ is $\psi_l(p_h) \equiv \frac{p_h s_l}{2}$ and since $\psi_l(.)$ always lies strictly in the region where firm **l** enjoys a positive market share, the low quality best reply function is :

$$\varphi_l(p_h) = \psi_l(p_h) \tag{A4}$$

As for the high quality firm, two regions are of interest : the monopoly region and the duopoly one. In the monopoly region ($p_l > p_h s_l$), the best reply is the monopoly price 1/2 which is feasible if and only if $p_l > s_l/2$. Otherwise Π_h is strictly increasing in the monopoly region and we always reach the duopoly region where the profit is

$$\Pi_h(p_h, p_l) = p_h D_h(p_h, p_l) = p_h \left[1 - \frac{p_h - p_l}{1 - s_l}\right] \tag{A5}$$

The solution to $\frac{\partial \Pi_h}{\partial p_h} = 0$ is $\psi_h(p_l) \equiv \frac{p_l + 1 - s_l}{2}$; it is interior to the monopoly area if $\psi_h(p_l) \leq \frac{p_l}{s_l}$ which holds true if and only if $p_l \leq \frac{s_l(1-s_l)}{2-s_l}$. Otherwise, $\Pi_h(.,p_l)$ is strictly decreasing in the duopoly region and the frontier price $\frac{p_l}{s_l}$ is optimal. As we have $\frac{s_l(1-s_l)}{2-s_l} < \frac{s_l}{2}$, the (kinked) best reply of firm **h** is

$$\varphi_h(p_l) = \begin{cases} \psi_h(p_l) & \text{if} \quad p_l \leq \frac{s_l(1-s_l)}{2-s_l} \\ \frac{p_l}{s_l} & \text{if} \quad \frac{s_l(1-s_l)}{2-s_l} \leq p_l \leq \frac{s_l}{2} \\ \frac{1}{2} & \text{if} \quad \frac{s_l}{2} \leq p_l \end{cases} \tag{A6}$$

As one can see on Figure 1 in the text, the free trade equilibrium $\left(p_l^*, p_h^*\right) = \left(\frac{s_l(s_h - s_l)}{4s_h - s_l}, \frac{2s_h(s_h - s_l)}{4s_h - s_l}\right)$ is given by the intersection of ψ_l and ψ_h.

A.2 Proof of Proposition 1

First, we define each firm's best reply. The frontier between the two regimes (Free Trade vs Quota) is found by equating the Free Trade foreign demand $1 - \tilde{x}$ with **q** and leads to the equation $p_d = \beta(p_f) \equiv p_f - (1-q)(1-s_d)$; the demands are therefore[17] :

$$D_d(p_d, p_f) = \begin{cases} \tilde{x} - x_d & \text{if } p_d \leq \beta(p_f) \\ 1 - q - x_d & \text{if } p_d > \beta(p_f) \end{cases} \tag{A7}$$

$$D_f(p_d, p_f) = \begin{cases} 1 - \tilde{x} & \text{if } p_d \leq \beta(p_f) \\ q & \text{if } p_d > \beta(p_f) \end{cases} \tag{A8}$$

Referring to Figure 4 in the text, in the "quota" region, the foreign firm faces a constant demand,

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[16]As will be discussed later on, this particular assumption of partial market coverage does not affect qualitatively our results.

[17] With regards to the Free Trade case, D_f remains concave and D_d still exhibits an outward kink but this has now a very different effect since the demand is not nil anymore beyond the kink.

thus increasing profits. It chooses the maximal price which is by definition the frontier price $\beta(p_f)$. Using the continuity of payoffs, we note that this price is itself dominated by the best reply of the "FT" region. The latter is $\psi_f(p_d) = \frac{p_d + 1 - s_d}{2}$ whenever it is attainable. The best reply of the foreign producer is displayed as a dashed bold line on Figure 4. It is continuous with a kink at $\bar{p}_d \equiv (2q-1)(1-s_d)$, the solution to $\psi_d(p_f) = \beta(p_f)$. Formally, we obtain :

$$\varphi_f(p_d) = \begin{cases} \psi_f(p_d) & \text{if } p_d \leq \bar{p}_d \\ \beta(p_d) & \text{if } p_d > \bar{p}_d \end{cases} \tag{A9}$$

The analysis is more involved for the domestic producer, because the optimal behaviour in the two regions are quite different. In the "quota" region, the domestic producer acts as a monopoly over a market of size $1-q$, thus the profit reaches a maximum of $\frac{s_d(1-q)^2}{4}$ at the "security" price $p_d^s \equiv \frac{(1-q)s_d}{2}$. This strategy is referred to as "hiding behind the quota".

In the Free Trade region, the best reply is $\psi_d(p_f) = \frac{p_f s_d}{2}$ which amounts to fight for market shares and yields a payoff increasing in p_f. It remains to choose between those two candidate best replies by solving :

$$\frac{s_d(1-q)^2}{4} = \Pi_d\big(\psi_d(p_f), p_f\big) = \frac{s_d p_f^2}{4(1-s_d)} \Leftrightarrow p_f = \mu(q,s_d) \equiv (1-q)\sqrt{1-s_d} \tag{A10}$$

To analyse the position of this benchmark and choose between p_d^s and $\psi_f(p_d)$, consider the pair of prices $(\tilde{p}_f, \tilde{p}_d)$ on Figure 4 at the intersection of $\beta(.)$ and $\psi_d(.)$. Because $\Pi_d(., \tilde{p}_f)$ is continuous and increasing over $\left[\tilde{p}_d, p_d^s\right]$ in the "quota" region, $\tilde{p}_d$ is dominated by p_d^s. It follows from this simple observation that $\mu(q,s_d) > \tilde{p}_f$ and that against a relatively low p_f, the domestic firm is inclined to use p_d^s whereas it fights for market shares against high foreign prices. This explains the shape of the best reply curve of the domestic producer displayed in bold on Figure 4. Formally, we obtain :

$$\varphi_d(p_f) = \begin{cases} p_d^s & \text{if } p_f \leq \mu(q,s_d) \\ \psi_d(p_f) & \text{if } p_f > \mu(q,s_d) \end{cases} \tag{A11}$$

Now, we are able to prove Proposition 1. Note that $\varphi_d(.)$ is discontinuous, so that we cannot ensure the existence of a pure strategy equilibrium. Because $\mu(q,s_d) > \tilde{p}_f$, the only candidate for a pure strategy equilibrium is the free trade equilibrium. For this equilibrium to exist, it must be true that $p_f^* < \mu(q,s_d) \Leftrightarrow q > q^*(s_d) \equiv 1 - \frac{2\sqrt{1-s_d}}{4-s_d}$ (a convex function increasing from 1/2 to 1). Otherwise, the equilibrium is in mixed strategies. However, the argument of concavity of the profit used in Lemma 1 still applies for the foreign firm which plays a pure strategy in equilibrium. Therefore, the only candidate for an equilibrium is $p_f = \mu(q,s_d)$ while firm **d** randomises between p_d^s and $\psi_d\big(\mu(q,s_d)\big)$, the weights over those two atoms being such that $\mu(q,s_d)$ is indeed a best reply against the mixture. ♦

A.3 Proof of Proposition 2

Recall first of Figure 7 which displays the four types of equilibria in the (q,s_f) space, under the normalisation $s_d = 1$. Recall that when the foreign firm sells the high quality, the domestic firm always recovers the rationed consumers and is therefore never excluded from the market. In the present context where $s_f < 1$, the possibility that Π_f is nil removes the underlying concavity property ; therefore the foreign firm may have multiple best replies and there is no unicity of the price equilibrium which may involve mixed strategies by both firms. This is key in understanding why proving Proposition 2 cannot be done by simply extending the argument of Krishna relative to the existence of a semi-mixed equilibrium.

i) In Area A, the Free Trade equilibrium still exists. The first existence condition is that the quota is not binding at the FTE level i.e., $q > D_f^* = \frac{1}{4-s_f}$ which defines the frontier between areas A and B. We claim that the other existence condition is $\rho(q,s_f) < p_f^* = \frac{s_f(1-s_f)}{4-s_f} \Leftrightarrow q > q^{A/C}(s_f) \equiv 1 - \frac{4(1-s_f)}{(4-s_f)\sqrt{1-s_f}}$. Indeed, as one can check by algebraic manipulations, $q^{A/C}(s_f) > \hat{q}(s_f)$ so that $\rho(q,s_f)$ is indeed the relevant price to compare with p_f^*. Area A can now be precisely defined by $q > \max\left\{q^{A/C}(s_f), \frac{1}{4-s_f}\right\}$. If the quota is set at the FTE level, it means that we are following the frontier A/B on Figure 7 and therefore, the quota is ineffective as long as s_f < 65,7% the solution of $q^{A/C}(s_f) = \frac{1}{4-s_f}$. The profits that obtain in this area are $\Pi_f^* = \frac{s_f(1-s_f)}{(4-s_f)^2}$ and $\Pi_d^* = \frac{4(1-s_f)}{(4-s_f)^2}$.

ii) In area B, the equilibrium is still in pure strategies but it is constrained. The foreign firm plays the maximum price compatible with sales of q i.e., $\gamma(.)$ while the domestic firm plays along its classical best reply $\psi_d(.)$. The equilibrium is thus $p_f^B = \frac{s_f(1-s_f)(1-2q)}{2-s_f}$ and $p_d^B = \frac{(1-s_f)(1-qs_f)}{2-s_f}$. The validity conditions for this equilibrium are that $\gamma(.)$ be the relevant cut-off i.e., $\bar{p}_d < p_d^B \Leftrightarrow q < \frac{1}{4-s_f}$ and $p_f^B > \rho(q,s_f) \Leftrightarrow q > q^{B/C}(s_f) \equiv \frac{(2-s_f)\sqrt{1-s_f} - 2(1-s_f)}{(2-s_f)\sqrt{1-s_f} - 2s_f(1-s_f)}$ which is also larger than $\hat{q}(s_f)$ meaning that ρ was the relevant price to compare with p_f^B. Area B is now defined by $\frac{1}{4-s_f} > q > q^{B/C}(s_f)$.

The profits are $\Pi_f^B = p_f^B q = \frac{qs_f(1-s_f)(1-2q)}{2-s_f}$ and $\Pi_d^B = p_d^B\left(1-\tilde{x}(p_f^B, p_d^B)\right) = \frac{(1-s_f)(1-qs_f)^2}{(2-s_f)^2}$. Notice that $\frac{\partial \Pi_f^B}{\partial s_f} = \frac{q(1-2q)\left((2-s_f)^2-2\right)}{(2-s_f)^2} > 0$ over area B.

iii) When **q** decreases, we leave area A or B to enter area C. One equilibrium sees the foreign firm playing the pure strategy $\rho(q,s_f)$ while the domestic one mixes over its security price p_d^s and the best reply $\hat{p}_d \equiv \psi_d(\rho(q,s_f))$. The weight α put on p_d^s is such that $\Pi_f^C = p_f\left[\alpha q + (1-\alpha)\left(\frac{\hat{p}_d s_f - p_f}{1-s_f}\right)\right]$ is indeed maximum at $p_f = \rho(q,s_f)$. The solution to $\frac{\partial \Pi_f^C}{\partial p_f} = 0$ is $p_f(\hat{p}_d, \alpha) = s_f\frac{\alpha q(1-s_f)+(1-\alpha)\hat{p}_d}{2(1-\alpha)}$. We then solve $p_f(\hat{p}_d,\alpha) = \rho(q,s_f)$ using $\hat{p}_d = \frac{\rho+1-s_f}{2}$ to get $\alpha(\rho) = \frac{(4-s_f)\rho-(1-s_f)s_f}{(1-s_f)(2q-1)s_f+(4-s_f)\rho}$ which is indeed a positive number less than unity. Finally, we obtain $\Pi_f^C(q,s_f) = \frac{2q.\rho(q,s_f)^2}{(1-s_f)(2q-1)s_f+(4-s_f)\rho(q,s_f)}$ and $\Pi_d^C(q,s_f) = \frac{(1-q)^2}{4}$. The lower contour of area C is simply $\hat{q}(s_f)$ as we are using the cut-off value $\rho(q,s_f)$.

iv) Area is formally defined by $q < \hat{q}(s_f)$. One could expect to find a simple mixed strategy equilibrium where the foreign firm plays $p_f = \lambda(q,s_f)$ instead of $\rho(q,s_f)$ and where the domestic one mixes over p_d^s with probability α and $\frac{\lambda(q,s_f)}{s_f}$ (cf. Figure 6a) with probability $1-\alpha$. However, this is not an equilibrium because $\Pi_f^D = p_f\left[\alpha q + (1-\alpha)D_f(p_f, \frac{\lambda}{s_f})\right]$ is increasing at $p_f = \lambda(q,s_f)$ for every possible

α since D_f is nil in region "M". Yet, the pricing game has continuous payoff functions and therefore possesses at least one equilibrium which must then involve completely mixed strategies F_f and F_d. Now, as shown in lemma 4 of Boccard & Wauthy [97b] for an extension of the present game, the equilibrium strategies involves no densities but only atoms.

Lemma 4 below shows that $\Pi_f^D(q,s_f)$ the equilibrium payoff of the foreign firm is bounded by $q.\lambda(q,1)$ which is precisely the equilibrium payoff of the foreign firm in the limiting case where $s_f = 1$ as analysed by Levitan & Shubik [72].

Lemma 4

In the completely mixed strategy equilibrium of the pricing game where the foreign firm is of low quality, its equilibrium payoff is bounded by the Levitan-Shubik limit.

Proof As shown in lemma 4 of Boccard & Wauthy [97b], the pricing equilibrium involves no densities but only atoms[18]. Let F_f and F_d be the distributions in equilibrium.

Claim 1 p_d $\text{Supp}(F_d), p_d \le \frac{1-q}{2}$.

Indeed, consider the largest atoms $\hat{p}_f$ and $\hat{p}_d$ played by the foreign and domestic firms. If the line (with respect to Figure 10) of points $(p_d, \hat{p}_f)_{p_d \ \text{Supp}(F_d)}$ belongs to area M or FT then $\Pi_f(.,F_d)$ is either nil or decreasing at $\hat{p}_f$, thus $\hat{p}_f$ cannot be part of an equilibrium. Therefore, the point $(\hat{p}_d, \hat{p}_f)$ must belong to areas NS or Q which means that the whole column $(\hat{p}_d, p_f)_{p_f \ \text{Supp}(F_f)}$ belongs to areas NS or Q.

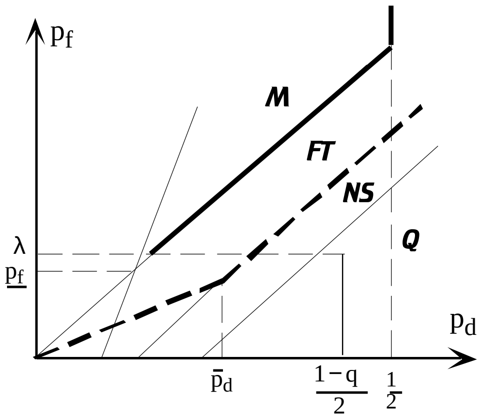

Figure 10

For points $(\hat{p}_d, p_f)$ belonging to area NS, $\Pi_d(.,p_f)$ is decreasing at $\hat{p}_d$ because the best reply is on the left in area FT. If $\hat{p}_d$ is larger than $\frac{1-q}{2}$, then $\Pi_d(.,p_f)$ is decreasing at $\hat{p}_d$ for all points $(\hat{p}_d, p_f)$ belonging to area Q. Combining the two results implies that the average $\Pi_d(.,F_f)$ is decreasing at $\hat{p}_d$, a contradiction to $\hat{p}_d$ being an atom of the equilibrium strategy of the domestic firm.

Claim 2 The equilibrium payoff $\Pi_f^D(q,s_f)$ is bounded by $q.\lambda(q,1)$

If $(\hat{p}_d, \hat{p}_f)$ belongs to area NS, $\hat{p}_f \le \hat{p}_d s_f - s_f(1-s_f)q$ and the domestic demand in equilibrium satisfies $D_d(F_f,F_d) \le 1 - \tilde{x}(\hat{p}_f, \hat{p}_d) = 1 - \frac{\hat{p}_d - \hat{p}_f}{1-s_f} \le \frac{1-s_f-\hat{p}_d+\hat{p}_d s_f - s_f q(1-s_f)}{1-s_f} = 1 - \hat{p}_d - s_f q$ (A12)

Hence $\Pi_d(F_f,F_d) \le \hat{p}_d\left(1 - s_f q - \hat{p}_d\right) \le \frac{1-q}{2}\left(1 - s_f q - \frac{1-q}{2}\right) \equiv U$ (A13)

If $(\hat{p}_d, \hat{p}_f)$ belongs to area Q then $D_d(\hat{p}_d, F_f) = \frac{1-q}{2}$ which is tighter than (A12). Now, if $\underline{p}_f > \lambda(q,1)$, the domestic firm enjoys a monopoly demand over $\left[0; \frac{p_f}{s_f}\right]$, so that $\Pi_d^D(q,s_f) \ge \frac{p_f}{s_f}\left(1 - \frac{p_f}{s_f}\right)$

[18] This is due first to the piecewise concavity of Π_f and Π_d in respectively p_f and p_d, and to the fact that both the nil price and any price above the monopoly one are strictly dominated for each firm.

$$> \frac{\lambda(q,1)}{s_f}\left(1-\frac{\lambda(q,1)}{s_f}\right) = \frac{1-\sqrt{q(2-q)}}{4s_f^2}\left(2s_f - 1 + \sqrt{q(2-q)}\right) \equiv L \qquad \text{(A14)}$$

This is true because the profit function is a parabola increasing at $\lambda(q,1)$. We get a contradiction by checking that L > U over area D (we solve for U = L and plot the solution that appears to lie above $\hat{q}(s_f)$). The claim is proved by observing that D_f is always less than **q** and that the equilibrium profit of firm **f** can be computed at any of the atoms it plays. In particular, at $\underline{p}_f$, $\Pi_f = D_f.p_f \leq q.\lambda(q,1)$. ♦

A.4 Proof of Lemma 2

When $q > q^*(s_d) \equiv 1 - \frac{2\sqrt{1-s_d}}{4-s_d}$, the Free Trade equilibrium prevails and we have already seen that the unconditional best reply of the low quality firm was 4/7. Thus, in our particular setting, the domestic best reply to $s_f = 1$ is 4/7 until we hit the lower contour of the free trade zone as can be seen on Figure 11. As for the foreign firm, its profit increases with its own quality and it shall always maintain the highest possible differential.

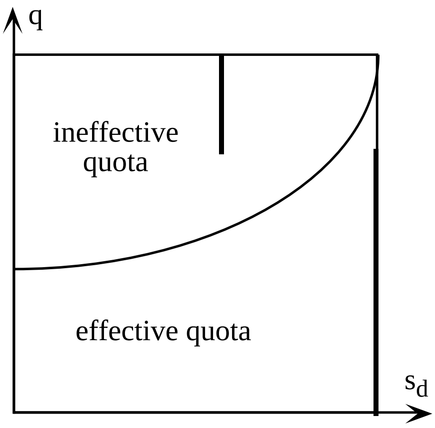

Figure 11

In the quota area, the unique equilibrium sees the foreign firm playing the pure strategy $F_f = \mu(q,s_d)$ while the domestic firm playing the mixed strategy F_d.

F_d is play p_d^s and $\psi_d(\mu(q,s_d)) = \mu(q,s_d)\frac{s_d}{2}$ with respective probabilities α and $1-\alpha$. Profits are:

$$\Pi_d^Q(s_d) = p_d^s\left(1 - q - \frac{p_d^s}{s_d}\right) = \frac{s_d(1-q)^2}{4} \qquad \text{(A15)}$$

$$\Pi_f^Q = p_f\left[\alpha q + (1-\alpha)\left(1 - \tilde{x}\left(p_f, \mu(q,s_d)\frac{s_d}{2}\right)\right)\right] \qquad \text{(A16)}$$

To understand formula (A4), one must recall that a mixed strategy gives the same payoff for any of its atoms. It is obvious from (A4) that the domestic profit reaches a maximum of $\frac{(1-q)^2}{4}$ for $s_d = 1$. It remains to compare it with the optimum of the Free Trade zone i.e., $\Pi_d^*(\frac{4}{7}) = \frac{1}{48}$. A simple algebraic computation shows that $s_d = 1$ is the overall optimal choice if and only if $q < \frac{2\sqrt{3}-1}{2\sqrt{3}} \approx 71\% > q^*(\frac{4}{7})$. The best reply is drawn in bold on Figure 11 above. Formally, we have :

$$BR_d(s_f / s_d \leq s_f) = \begin{cases} s_f & \text{if } q \leq 71\% \\ \frac{4}{7}s_f & \text{if } q > 71\% \end{cases} \qquad \text{(A17)}$$

As for the foreign firm, we need to compute precisely the mixed strategy equilibrium. The optimal foreign price is solution of :

$$\frac{\partial \Pi_f^Q}{\partial p_f} = 0 \Leftrightarrow p_f = \frac{2(1-s_d)(1+\alpha q - 2\alpha) + (1-\alpha)s_d\mu(q,s_d)}{4(1-\alpha)} \qquad \text{(A18)}$$

In equilibrium $p_f = \mu(q,s_d)$

$$\alpha = \frac{2(1-s_d) + \mu(q,s_d)(s_d-4)}{2(1-s_d)(1-q) + \mu(q,s_d)(s_d-4)} \qquad \text{(A19)}$$

Plugging into (A5) $\Pi_f^Q = \frac{2q\mu(q,s_d)^2}{2(1-s_d)(1-q)+\mu(q,s_d)(s_d-4)}$ (A20)

Using $\mu(q,s_d) = (1-q)\sqrt{1-s_d}$ $\Pi_f(q,s_d) \equiv \frac{2q(1-q)(1-s_d)}{(4-s_d)\sqrt{1-s_d}-2(1-s_d)}$ (A21)

After some algebraic manipulations, we can show that $\frac{\partial \Pi_f}{\partial s_d}$ is proportional to $s_d + s_d^2 - 2$ which is negative as $s_d < 1$. Recalling now that s_d stands for $\frac{s_d}{s_f}$, we can conclude that, as in the free trade zone, the foreign firm's profit increases with its own quality, we have : $BR_f(s_d / s_d \le s_f) = 1$. ♦

A.5 Proof of Lemma 3

Using the upper bound $\Pi_f^D(q,s_f) = q.\lambda(q,1)$ derived in lemma 0 above, we plot $\Pi_f(q,s_f)$ on Figure 12 below over the four areas A, B, C and D that are indicated by different shades of grey. In area A, the Free Trade equilibrium prevails and we have already seen that the unconditional best reply of the low quality firm was $s_f = 4/7$ leading to a profit of 1/48. In areas B and C, $\Pi_f(q,s_f)$ is a smooth concave and increasing function of quality thus we enter area D. There, the equilibrium profit is less than $q.\lambda(q,1)$, the profit accruing to the foreign firm when $s_f = 1$ as in the Levitan & Shubik [72] model. The solution to $q.\lambda(q,1) = 1/48$being approximately 0.65, we obtain :

$$BR_f(s_d / s_f \le s_d) = \begin{cases} s_d & \text{if } q \le 65\% \\ \frac{4}{7}s_d & \text{if } q > 65\% \end{cases} \quad \text{(A22)} \ ♦$$

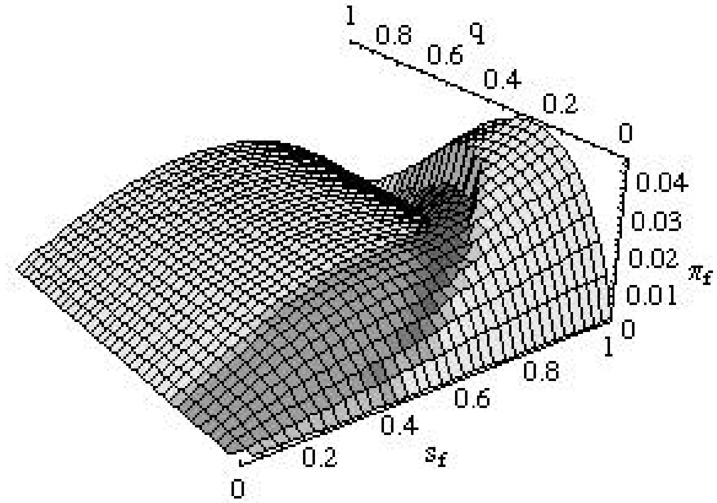

Figure 12

A.6 Proof of Proposition 3

Consider first $q < \underline{q}$ and a candidate equilibrium pair of qualities such that $s_d > s_f$. We know from Lemma 3 that the foreign firm will, at least, want to imitate the domestic one and will set $s_f = s_d$. We are now in the area where $s_d \ge s_f$ and Lemma 2 tells us that the foreign firm wishes to maximise its quality level ($s_f = 1$). We use Lemma 2 again: for $q < \underline{q} < \bar{q}$, the domestic firm wish to imitate the foreign one. The case for $s_d \le s_f$ being symmetric we have shown that the only pure strategy equilibria is $s_d = s_f = 1$ i.e., **homogeneous products**.

Consider then a quota such that $\underline{q} < q < \bar{q}$. Starting from $s_f > s_d$, the domestic firm will imitate the foreign one (Lemma 2). Being in the area where $s_d \geq s_f$, the foreign firm will choose to differentiate to $s_f = \frac{4}{7} s_d$ because **q** is larger than $\underline{q}$ (Lemma 3). For a quota between $\underline{q}$ and $\bar{q}$ and a quality level s_f in the neighbourhood of $\frac{4}{7} s_d$, the equilibrium is the Free Trade one. Thus, the first period domestic profit is $\Pi^*_d(s_f, s_d) = \frac{4s_d(s_d - s_f)}{(4s_d - s_f)^2}$ which is increasing with s_d so that the domestic firm will always increase a little its quality while the foreign one will always maintain an optimal ratio of 4/7. The unique equilibrium is therefore $s_d = 1$ and $s_f = 4/7$.

Lastly, for $q > \bar{q}$, both firms choose to differentiate to an optimal ratio of 4/7 when they are of low quality which means that $s_f = 1$ and $s_d = 4/7$ is a second asymmetric equilibrium, symmetric to the previous one: $s_f = 4/7$ and $s_d = 1$.[19] ♦

A.7 Proof of Proposition 4

Robustness of the equilibria and optimal quota with respect to positive cost for quality

As Lemmas 2 and 3 use on the Free Trade equilibrium, we first solve for the optimal choices under Free Trade. From $\Pi^{FT}_l(F, s_h, s_l) = \frac{s_h s_l (s_h - s_l)}{(4s_h - s_l)^2} - \frac{s_l^2}{F}$ we obtain a FOC which has a unique real third degree polynomial solution $x(F, s_h)$. Using $\Pi^{FT}_h(F, s_h, s_l) = \frac{4s_h^2(s_h - s_l)}{(4s_h - s_l)^2} - \frac{s_h^2}{F}$ we find the equilibrium by solving numerically $\left. \frac{\partial \Pi^{FT}_h}{\partial s_h} \right|_{s_l = x(F, s_h)} = 0$ in s_h. We obtain an increasing function $s^*_h(F)$ converging to unity at F = 8 (cf. Boccard & Wauthy [99]). Likewise $s^*_l(F) \equiv x\left(F, s^*_h(F)\right)$ is increasing concave and converges to 4/7 as shown on Figure 13a. The equilibrium payoff $\Pi^*_l(F)$ is increasing concave and converges to $1/48 \approx 0.021$ while $\Pi^*_h(F)$ shown on Figure 13b is rapidly increasing while s^*_l is low but decreases as s^*_l converges to 4/7.

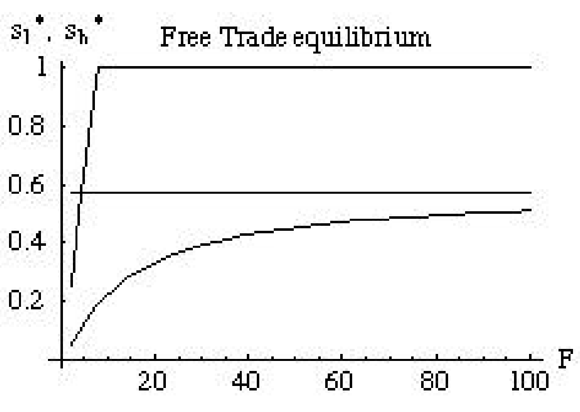

Figure 13a

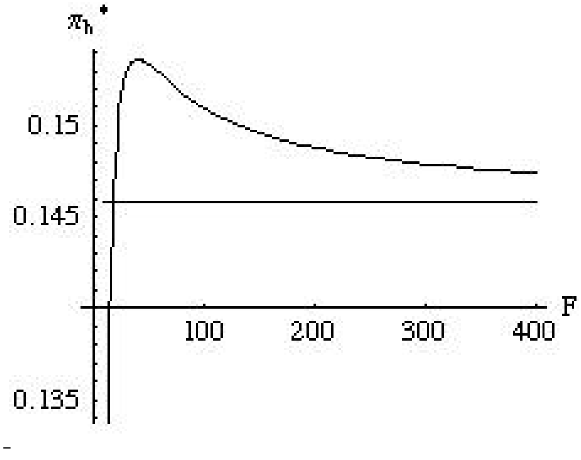

Figure 13b

[19] Let us stress again that our conclusion of no differentiation in equilibrium does not hinge on any assumption made about the assignment of low and high quality to either the domestic or the foreign firm. What we are talking about is the subgame perfect equilibrium in a game where qualities are selected simultaneously without assuming any a priori hierarchy.

The extension of Lemma 2 involves the payoff in the quota zone $\Pi_d^Q(F,s_d) = \frac{s_d(1-q)^2}{4} - \frac{s_d^2}{F}$ and the optimal quality is $s_d^Q(F,q) = \mathrm{Min}\left\{1, \frac{F}{8}(1-q)^2\right\}$. It is valid if the point $\left(s_d^Q(F,q), q\right)$ lies in the quota area of Figure 11 above i.e., if $q \le q^*\left(s_d^Q(F,q)\right) \Leftrightarrow F \ge 16\frac{2(1-q)^2 - 1 + \sqrt{1-3(1-q)^2}}{(1-q)^4} \Leftrightarrow q \le \delta(F)$ where δ is a polynomial increasing concave function with $\delta(0) = \frac{1}{2}$ and $\delta(+\infty) = 1$. When $q > \delta(F)$ the optimal quality is $s_d = q^{*-1}(q)$. The optimal quality leads to $\Pi_d^Q(F,q) = \frac{F}{64}(1-q)^4$ when $q \le \delta(F)$.

The cut-off between imitation and differentiation is the solution $\bar{q}(F)$ of $\Pi_d^Q(F,q) = \Pi_l^*(F)$; it is lesser than $\delta(F)$ for F > 7.35 (this is necessary for the above comparison to be meaningful), increasing concave and converges to 71% (cf. Figure 9 in the text). Furthermore $s_d^Q(F,\bar{q}(F)) = \frac{F}{8}\left(1-\bar{q}(F)\right)^2$ reaches the top quality at F = 61 so that the quality jump $s_d^Q(F,\bar{q}(F)) - s_l^*(F)$ occurring when the quota passes below $\bar{q}(F)$ reaches a maximum at F = 61 and then decreases to its limit 3/7 (cf. Figure 14 below). We have thus proven that for $q < \bar{q}(F)$ (and F > 8), the domestic firm will not differentiate as under "Free Trade" but will instead leapfrog to a quality closer to the foreign one.

If $q > \bar{q}(F)$ the equilibrium is the Free Trade one with $s_d = s_l^*(F)$ and $s_f = s_h^*(F)$. Otherwise the domestic firm chooses $s_d^Q(F,q)$. The foreigner payoff is $s_f \Pi_f\left(q, \frac{s_d}{s_f}\right) - \frac{s_f^2}{F}$. When computed at $s_d = s_d^Q(F,q)$ it is increasing in s_h as soon as F > 8, thus the foreigner firm always choose top quality whatever the quota.

The extension of Lemma 3 involves area D where $\Pi_f^D(q,s_f) = s_f q \frac{1-\sqrt{q(2-q)}}{2} - \frac{s_f^2}{F}$. The optimal quality is $s_d^D(F,q) = \frac{Fq\left(1-\sqrt{q(2-q)}\right)}{4}$ which is valid if larger than $\hat{q}^{-1}(q)$. This can never happen when F > 32 indicating that the best reply would have to be found in areas C or B. Computations indicate that this solution is always dominated by the optimal Free Trade differentiation (they are all quite small since cost is relatively large). When F > 32 the optimal payoff associated with "imitation" is $\Pi_f^D(F,q)$. The cut-off between imitation and differentiation is the solution in $\underline{q}(F)$ of $\Pi_f^D(F,q) = \Pi_l^*(F)$. It is an increasing concave and converges to 65% (cf. Figure 9 in the text) and as seen in the above paragraph, the best reply $s_d^D\left(F,\underline{q}(F)\right)$ reaches the top quality nearby F = 61. Figure 14 displays the quality jumps that occur when the quota passes below $\bar{q}(F)$ and $\underline{q}(F)$; they are almost identical and converge to $3/7 = 1 - 4/7$.

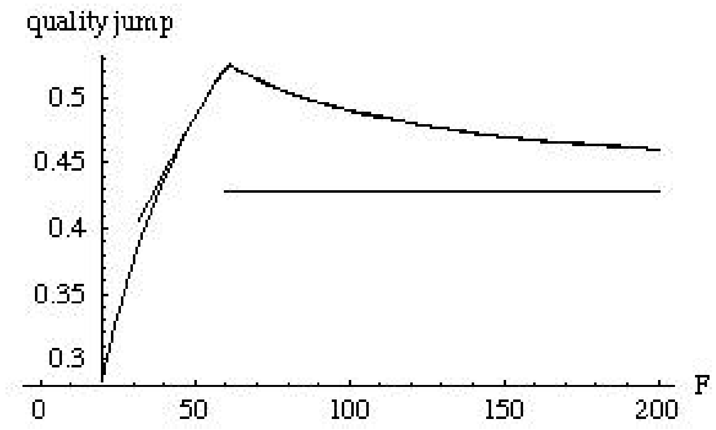

Figure 14

A.8 Proof of Proposition 5

We compute the consumer surplus in the Levitan & Shubik [72] model where both firm have chosen top quality. These authors show that in equilibrium, firms play a mixed strategy with support $\left[\lambda(q,1);\frac{1-q}{2}\right]$ and cumulative distributions $F_d(p) = 1-\frac{\lambda(q,1)}{p}$, $F_f(p) = \frac{p(1-p)-\lambda(q,1)(\lambda(q,1)-1)}{pq}$. As $F_f(\lambda(q,1)) = 0$, $F_f\left(\frac{1-q}{2}\right) = 1$, $F_d(\lambda(q,1)) = 0$ and $F_d\left(\frac{1-q}{2}\right) < 1$, only the domestic firm has an atom at $\frac{1-q}{2}$.

The surplus of the consumer located at **x** is best understood by separating two cases

- if $x > p_d$, the foreign price p_f is the lowest with probability $F_f(p_d)$ in which case the consumer buy at the price p_f (because $x > p_d > p_f$) so that we need to compute an expectation. With complementary probability, the consumer buys at the domestic firm, thus the consumer surplus is:

$$H(x,p_d) = (x-p_d)\left(1-F_f(p_d)\right) + \int_{\lambda(q,1)}^{p_d}(x-p_f)dF_f(p_f) \qquad \text{(A23)}$$

- if $x < p_d$, then the consumer surplus becomes $G(x,p_d) = \int_{\lambda(q,1)}^{x}(x-p_f)dF_f(p_f)$

Integrating with respect to the distribution of domestic prices, we have again two cases according to the respective positions of **x** and the upper price limit:

- if $x < \frac{1-q}{2}$: $\underline{W}(q,x) = \int_{\lambda(q,1)}^{x} H(x,p_d)dF_d(p_d) + \int_{x}^{\frac{1-q}{2}} G(x,p_d)dF_d(p_d) + G(x,\frac{1-q}{2})\left(1-F_d(\frac{1-q}{2})\right)$

- if $x > \frac{1-q}{2}$: $\overline{W}(q,x) = \int_{\lambda(q,1)}^{\frac{1-q}{2}} H(x,p_d)dF_d(p_d) + H(x,\frac{1-q}{2})\left(1-F_d(\frac{1-q}{2})\right)$

Integrating with respect to the uniform distribution of consumer over the range of potential buyers i.e., $x \geq \lambda(q,1)$, we get the consumer surplus:

$$W_C^Q(q) = \int_{\lambda(q,1)}^{\frac{1-q}{2}} \underline{W}(q,x)dx + \int_{\frac{1-q}{2}}^{1} \overline{W}(q,x)dx \qquad \text{(A24)}$$

The domestic surplus $W_d^Q(q) = W_C^Q(q) + \frac{(1-q)^2}{4}$ is plotted on Figure 15.[20] One can observe that $W_d^Q(q)$ is concave, increasing and tends towards 1/2 at the free trade limit (q = 1) i.e., the surplus in the polar case of Bertrand competition without quotas.

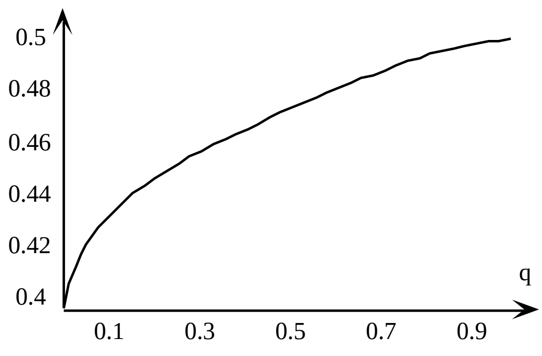

Figure 15

If the quota is set to a level larger than 65% but lesser than 71%, the foreign firm will differentiate to 4/7 and in the pricing sub-game, firms play the Free Trade pure strategy equilibrium $\left(p_f^* = \frac{1}{14}, p_d^* = \frac{1}{4}\right)$.

[20] The exact formula is available upon request from the authors.

The optimal demands are thus $\left(D_f^* = \frac{7}{24}, D_d^* = \frac{7}{12}\right)$ and the profits $\left(\Pi_f^* = \frac{1}{48}, \Pi_d^* = \frac{7}{48}\right)$. The domestic surplus is easily computed as

$$W_d^{FT} = \int_{1-D_d^*}^{1} (x - p_d^*)dx + \int_{1-D_d^*-D_f^*}^{1-D_d^*} \left(\tfrac{4}{7}x - p_f^*\right)dx + \Pi_d^* = \tfrac{7}{16} \approx 0.438 \qquad \text{(A25)}$$

Setting a looser quota (i.e., q > 71%) would yield either the same surplus or an even smaller one if the quality choice equilibrium sees the domestic firm differentiating and thus earning less profits. As one can check from Figure 15 above, $W_d^Q(.65) \approx 0.48 > W_d^{FT}$, thus the optimal choice for the domestic government is to set a quota at the left hand limit of 65%.

We now turn to the foreign firm. For quotas lesser than 65%, its payoff is the Levitan & Shubik profit $\Pi_f^{LV} = \frac{q}{2}\lambda(q,1)$ which reaches its maximum $\frac{(\sqrt{2}-1)^2}{4} \approx 0.043$ for $q = 1 - \frac{1}{\sqrt{2}} \approx 29\%$. For 65% < q < 71%, the foreign firm differentiates and thus obtains $\frac{1}{48} \approx 0.021$. However, for q > 71%, the foreign firm might get $\frac{7}{48} \approx 0.146$ if it is the domestic firm who differentiates. It is then natural to consider the correlated equilibrium with equal probabilities on the asymmetric equilibria $\left(\frac{4}{7},1\right)$ and $\left(1,\frac{4}{7}\right)$. In such a case, the foreign firm gets on expectation $\frac{7+1}{2\times 48} \approx 0.08$. This latter payoff is the dominant choice so that the foreign firm comes to propose a quota that will never bite.

In the presence of cost for quality, the domestic surplus is less than $W_d^Q(q)$ but there is a bigger drop of quality when passing above $\underline{q}(F)$ as shown on Figure 14 above. Hence our preceding conclusion remains valid. Furthermore we have been able to verify that for F > 40, the optimal quota for the government $\underline{q}(F)$ is larger than the almost constant market share of the foreign firm under the Free Trade regime which at the limit $F = +\infty$ is equal to $\frac{1}{2}\left(\frac{7}{24} + 2\frac{7}{24}\right) = \frac{7}{16}$.♦

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