Ensuring Quality Provision in Deregulated Industries

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Abstract

The deregulation of public monopolies has often generated a decrease in quality and reliability of service. Governments have coped with this issue by imposing *Minimum Quality Standards* (MQS) to entrants but a recent stream of literature has raised concerns about the inadequacy of this instrument. We propose an alternative, a *sales quota* to be imposed on the incumbent, to overcome the competition effects that tend to generate quality downgrading. In our model an entrant invests into quality because the limit on the incumbent sales eliminates price wars and enable him (as well as the incumbent) to recoup investments in quality. The maximal welfare is obtained when the *sales quota* is fixed at 71% of the initial market. *Laisser-faire* leads to entrant's differentiation and a welfare of 91, 6% of the Pareto optimum while our solution leads to a welfare of 99, 4%. To reach this level a MQS should be set 66% above the ideal level chosen by the entrant under laisser-faire.

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1 Introduction

According to Rhoades & Waguespacks (2000) (and the reference therein) quality has sharply decreased after the deregulation of the US airline industry. Likewise the reliability of electricity provision in California has crumbled last year. The same observation applies even more dramatically for railway transportation in England. More generally, a government wishing to deregulate a monopolistic industry almost always faces the problem of ensuring that quality and reliability of the good or service remains high enough.

At first sight, one could think that competition per se is apt to increase product quality. Indeed lower quality products should not resist the challenge of higher quality ones entering the market. Nevertheless differentiation enables several products to coexist in a deregulated market. Choi, Lee & Cheng (2001) provide a recent case study in the Taiwan mobile telecommunication industry where selling a lower quality product at a lower price appears to be an optimal strategy. Further the vertical differentiation literature has shown that entry often reduces average quality and welfare (e.g., Crampes & Hollander (1995)). The argument is that a fierce price competition lowers firms revenues, thus to keep profit margins at the market level firms save on cost by disinvesting in quality.[1] The scenario occurring after deregulation is then very simple. Given that the incumbent has accumulated a high quality of service thanks to past public investments, entrants deliberately choose a lower, and thus cheaper, quality. Average quality then decreases and since the incumbent's profits also decreases it is very likely to downgrade quality in the future.[2]

As a reaction governments often condition entry to quality requirements, through contracts or more broadly by imposing Minimum Quality Standards (MQS).[3] Beyond the obvious agency problems relating to certification and monitoring of MQS, this instrument succeeds mainly in increasing the lowest of qualities. This may not be enough to ensure an increase in average quality because of the general tendency to lower quality. As shown by Lutz et al. (2000), the differentiation possibilities are reduced by the MQS and ultimately lower profits in the industry making it more difficult for an entrant to recoup the fixed cost of entry. Hence this policy may end up limiting entry and it is no surprise that monopolies favor such measures in the name of consumer protection. Maxwell

[1]Other compelling reasons pertain to contractibility issues and public good-like feature of quality. Auriol (1998) provides a very interesting analysis of such issues.
[2]Herguera & Lutz (1998) survey these issues within the context of international trade but the results they report apply to wider domains.
[3]As shown by Cremer & Thisse (1992) ad valorem taxes also could be used.

(1998) takes a different point of view to show that MQS may lead to less innovation and therefore to welfare losses. As a result of equilibrium interactions MQS tend to "hurt" high quality providers so that the ultimate effect of MQS seems to be a downgrade of quality from the top, especially in the long run.

The general conclusion we can draw from this literature is that MQS may not be optimal instruments. In order to ensure quality provision while allowing for more competition, governments could instead use an instrument which "compensates" firms in terms of incentives for upgrading quality to counter balance the negative effect on profits of the fierce competition at the pricing stage. In the present paper, we put forward such an instrument. It is quite simple and easy to implement as it amounts to limit from the outset the sales of one firm (typically the historical incumbent). We show within a duopoly game that a *sales quota* applied to the incumbent firm is sufficient to induce both firms to choose higher qualities in equilibrium, as compared to *laisser-faire*.

The intuition for our result is the following: imposing a sales quota on one firm drastically weakens the incentive to differentiate products at the quality choice stage because it dramatically relaxes competition at the pricing stage and enable firm to secure fair profit margins. When quality is not costly, the sales quota fosters minimal differentiation in the sense that both firms choose the best available quality and therefore end up selling homogeneous goods in equilibrium. When quality is costly, firms may or may not choose identical qualities in equilibrium; nevertheless, average quality increases and the degree of differentiation decreases (as compared to laisser-faire). Whether quality costs are small or not, the public authority finds it optimal to induce this "quality upgrading effect" with the help of the sales quota.

The argument is formally established in a three stage game. In the first stage, a government chooses the (optimal) sales quota to be imposed on the incumbent firm. In the second stage the entrant selects its quality, assuming that the incumbent is already committed to a high quality. Then price competition takes place at the third stage. We characterize the unique subgame perfect equilibrium of this game and show that both firms sell the (same) high quality product. Let us stress that the mechanism at work in our model is robust to more general settings. In particular, our results qualitatively generalize to cases where the government imposes a sales quota on the entrant as well as to cases where qualities are chosen simultaneously by the firms. We have retained the particular sequence described above only for the sake of simplifying the exposition.

The paper is organized as follows. In section 2 we recall the outcome of vertical differentiation under laisser-faire. In section 3 we solve for price competition when the

incumbent is constrained by a sales quota, then we go backward in the game tree by considering quality choices in section 4. Section 5 derives the optimal sales quota and compares our instrument to QMS before concluding.

2 Entry under Laisser-faire and Price Competition

In this article we put the emphasis on price competition because most of deregulations are done with the announced aim of lowering consumer prices thereby accrediting the idea that price is the main channel of competition in these markets. In this section we recall how quality downgrading may occur within the vertical differentiation model of Mussa & Rosen (1978).

Let us consider a two-stage game between an entrant and an incumbent. The incumbent's quality being already high the entrant must decide in the first stage if he wants to meet or differentiate downward. In the second stage, the two firms sell indivisible goods differentiated by their quality indexes s_l and s_h satisfying $s_l < s_h$. Firms produce at zero cost and maximize profits by setting prices p_l and p_h non-cooperatively. Consumers are willing to buy at most one unit of the good and exhibit heterogeneous preferences. They are identified by their taste for quality x which is uniformly distributed on the interval $[0;1]$. The net utility of consuming good $i \in \{l,h\}$ for the consumer with taste x is $u_i = xs_i - p_i$ and we set the default utility of no-consumption to zero.

To address the issue of quality choices in the first stage we assume that firms choose qualities at zero cost. We shall later test the robustness of this hypothesis by introducing a convex sunk cost. As a direct consequence of zero cost for quality we have to assume that the range of possible qualities is bounded with $s_i \in [0;1]$. Assuming that the incumbent has already chosen the best available quality, the subgame perfect equilibrium (in pure strategies) of the game described above is characterized in Lemma 1.

Lemma 1 *When quality is not costly, the entrant optimally differentiates to a ratio of $\frac{4}{7}$. The price equilibrium of the continuation game is unique and in pure strategies.*

Choi & Shin (1992) provide a proof of this standard result. However, recalling the essence of price competition under vertical differentiation will be useful for the analysis to follow. Figure 1 below depicts firms' best replies in a pricing game associated with quality choices $s_l < s_h = 1$. The price space can be partitioned in the *duopoly* region where both firms enjoy a positive market share and the *monopoly* region where only the high quality firm enjoys a positive demand. Since products exhibit different qualities, the low quality firm must offer a sufficient discount with respect to the high quality price to compensate for the quality differential. The required discount defines the frontier $p_l = p_h s_l$ between the monopoly and duopoly regions.

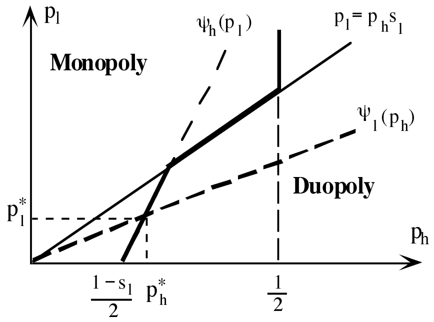

Figure 1

Obviously, the low quality firm's best reply (denoted by $\psi_l(p_h)$) lies in the interior of the duopoly region. In contrast, the best reply of the high quality firm is in the duopoly region only against low p_l. Against higher prices, the high quality firm optimally excludes the low quality product and enters into the monopoly region. Either by naming the limit price which is just sufficient for this purpose (in which case its best reply is at the frontier between the two regions) or by naming the monopoly price(which is $\frac{1}{2}$ in the present case).

As for quality choices, the intuition is well-known: choosing the same quality as the incumbent can only result into marginal cost pricing in equilibrium whereas differentiating allows for positive profits. Against a high quality, the entrant therefore optimally differentiates to relax price competition. Note that the differentiation argument is qualitatively independent of costs for quality (see Motta (1993)). As we show in Lemma 2 of the appendix, adding a convex cost for quality $\frac{s^2}{F}$ yields even more extreme results. For $F < 8$, the incumbent choice is $s_i^* \simeq \frac{F}{8}$ while the entrant chooses $s_e^* \simeq \frac{F}{42}$ thus whenever the incumbent has not chosen the maximal quality ($F < 8$) the entrant differentiates at a very low ratio of $\frac{8}{42} \simeq 0.19$. As the cost for quality becomes lower ($F \nearrow$) the incumbent selects the best available quality[4] while the entrant differentiates at a ratio increasing towards $\frac{4}{7} \simeq 0.57$ (at the limit $F = +\infty$).

Hence taking into account the cost of acquiring quality only reinforces differentiation and the potential concern of the government for quality downgrading. Further the literature on quantity competition has shown that quality differentiation appears only when

[4]The upper bound $s = 1$ should be seen as the ultimate technological advance.

it allows firms to save on (sunk) costs. The general conclusion under laisser-faire is that cost for quality and price competition are the factors leading to quality differentiation and average downgrading.

3 The Effect of a Sales Quota on Price Competition

Imposing a sales quota q to a firm is identical to assume that its production capacity is limited. Hence a Bertrand-Edgeworth analysis is called for since the incumbent may face a demand exceeding its capacity. When the entrant's price is very large relative to the incumbent's one ($p_e \gg p_i$) some consumers are rationed by the incumbent and may wish to transfer their purchase on the entrant.

This mere fact dramatically alters the nature of price competition because it allows for a new strategy profile. The entrant could indeed find it profitable to name a high price, anticipating that some consumers will be rationed by the incumbent and thus be recovered. This strategy amounts to enjoy a high unit margin on a limited market share. The profitability of this strategy (relative to the more standard aggressive strategy) depends on the propensity of rationed consumers to transfer their purchase to the entrant instead of refraining from consuming, i.e. on the importance of the spillovers, which in turn depend on who the rationed consumers are and on the substitutability of the goods.

This Bertrand-Edgeworth argument has been extensively developed in markets for homogeneous goods and it is well-known that equilibrium outcomes are heavily dependent on the rationing rule retained for the analysis (see in particular Davidson & Deneckere (1986)). Most papers however retain the so-called *efficient* rationing rule (see Deneckere et al. (2000)) and we shall adopt a comparable rationing rule by endorsing the interpretation put forward by Tirole (1988): rationed consumers are those who exhibit the lowest taste x for the product. In our framework a consumer wishing to buy the high quality product but rationed by the incumbent always prefers to buy the low quality product of the entrant instead of refraining from consuming. Thus if at the prevailing prices the demand addressed to the incumbent exceeds the quota q all rationed consumers are recovered by the entrant who faces a residual market $1 - q$ over which it maximizes profits.[5]

Krishna (1989) provides a characterization of equilibrium pricing in a similar setting. Our analysis can be viewed as an application of her methodology to vertical differentia-

[5]In the present case, the residual demand is in fact independent of the particular rationing rule. Note that a similar conclusion does not apply for the alternative quality hierarchy.

tion. The space of prices can be divided in the *binding* and the *non-binding* regions. As we show afterwards there exists a function $\beta(.)$ such that when the entrant price p_e is larger than $\beta(p_i)$ the demand addressed to the incumbent exceeds its capacity. There are thus two competition regimes. In the binding regime ($p_e > \beta(p_i)$), the entrant recovers all rationed consumers and becomes a monopoly over a restricted market of size $1-q$. The optimal price p_e^s is independent of the incumbent's one and is referred to as the "security" strategy (it yields a minimax payoff). As for the incumbent, sales are constant and equal to the capacity q thus the optimal price is $p_e = \beta(p_i)$. In the other regime the traditional Bertrand analysis applies: firms fight for market shares and get low profits whenever one price is low.

The crucial point then is to observe that the entrant will choose security over aggressiveness against an aggressive (low) p_i and the contrary if p_i is large. The switch from one regime to the other take place at the price μ where the entrant is indifferent between the two options. In contrast, the incumbent's best reply is continuous. The discontinuity in the entrant best reply precludes the existence of a pure strategy equilibrium for many parameter constellations but in the present setting, we can apply the equilibrium characterization proposed by Krishna (1989) to obtain the following proposition.

Proposition 1 *Assume $s_e < s_i = 1$. The price equilibrium is unique and there exists a threshold quota $\bar{q}(s_e)$ such that*
► *if $q > \bar{q}(s_e)$, the laisser-faire equilibrium prevails*
► *if $q \leq \bar{q}(s_e)$, the entrant randomizes over p_e^s and some lower price while the incumbent plays a pure strategy.*

Proof The frontier between the two price regimes is found by equating the incumbent $1-\tilde{x}$ with the sales quota q. It leads to the equation $p_e = \beta(p_i) \equiv p_i - (1-q)(1-s_e)$. The demands are therefore

$$D_i(p_e,p_i) = \begin{cases} 1-\tilde{x} & \text{if } p_e \leq \beta(p_i) \\ q & \text{if } p_e > \beta(p_i) \end{cases} \quad (1)$$

and

$$D_e(p_e,p_i) = \begin{cases} \tilde{x}-x_e & \text{if } p_e \leq \beta(p_i) \\ 1-q-x_e & \text{if } p_e > \beta(p_i) \end{cases} \quad (2)$$

In the binding regime, the incumbent faces a constant demand, thus increasing profits. It chooses the maximal price which is by definition the frontier price $\beta(p_i)$. Using the continuity of payoffs, we note that this price is itself dominated by the best reply of the

non binding regime. The latter is $\psi_i(p_e) = \frac{p_e+1-s_e}{2}$ whenever it is attainable. The best reply of the incumbent is continuous with a kink at $\bar{p}_e \equiv (2q-1)(1-s_e)$, the solution to $\psi_e(p_i) = \beta(p_i)$. Formally, we obtain

$$\phi_i(p_e) = \begin{cases} \psi_i(p_e) & \text{if } p_e \leq \bar{p}_e \\ \beta(p_e) & \text{if } p_e > \bar{p}_e \end{cases} \quad (3)$$

The analysis is more involved for the entrant because the optimal behavior in the two regimes are quite different. In the binding regime, the entrant acts as a monopoly over a market of size $1-q$, thus the profit reaches a maximum of $\frac{s_e(1-q)^2}{4}$ at the security price $p_e^s \equiv \frac{(1-q)s_e}{2}$. In the non binding regime, the best reply is $\psi_e(p_i) = \frac{p_i s_e}{2}$ which amounts to fight for market shares and yields a payoff increasing in p_i. It remains to choose between those two candidate best replies by solving $\frac{s_e(1-q)^2}{4} = \Pi_e(\psi_e(p_i),p_i) = \frac{s_e p_i^2}{4(1-s_e)} \Leftrightarrow p_i = \mu(q,s_e) \equiv (1-q)\sqrt{1-s_e}$.

To analyze the position of this benchmark and choose between p_e^s and $\psi_f(p_d)$, consider the pair of prices $(\tilde{p}_i,\tilde{p}_e)$ at the intersection of $\beta(.)$ and $\psi_e(.)$ on Figure 2 below.

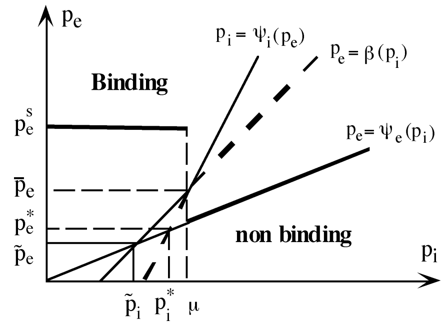

Figure 2

As $\Pi_e(.,\tilde{p}_i)$ is continuous and increasing over $[\tilde{p}_e,p_e^s]$ in the binding regime, $\tilde{p}_e$ is dominated by p_e^s. It follows from this simple observation that $\mu(q,s_e) > \tilde{p}_i$ and that against a relatively low p_i, the entrant is inclined to use p_e^s whereas it fights for market shares against high incumbent prices. Formally, we obtain

$$\phi_e(p_i) = \begin{cases} p_e^s & \text{if } p_i \leq \mu(q,s_e) \\ \psi_e(p_i) & \text{if } p_i > \mu(q,s_e) \end{cases} \quad (4)$$

Note that $\phi_e(.)$ is discontinuous, so that we cannot ensure the existence of a pure strategy equilibrium. Because $\mu(q,s_e) > \tilde{p}_i$, the only candidate for a pure strategy equilibrium is the laisser-faire equilibrium (p_i^*, p_e^*). For this equilibrium to exist, it must be true that (unlike on Figure 2 above) $p_i^* < \mu(q, s_e) \Leftrightarrow q > \bar{q}(s_e) \equiv 1 - \frac{2\sqrt{1-s_e}}{4-s_e}$ (a convex function increasing from $\frac{1}{2}$ to 1). Otherwise, the equilibrium is in mixed strategies and the only candidate for an equilibrium is $p_i = \mu(q, s_e)$ while the entrant randomizes between p_e^s and $\psi_e(\mu(q, s_e))$, the weights over those two atoms being such that $\mu(q, s_e)$ is indeed a best reply against the mixture. ■

Notice that if the entrant were facing a sales quota the analysis would be more involved because of additional spillovers effects. However, the qualitative conclusions of the present section would be preserved: either the laisser-faire equilibrium holds (for loose quotas and high quality differentials) or the non-constrained firm uses a mixed strategy in equilibrium that pays her according to the size of the residual market i.e., $1-q$. Since the equilibrium identified in Proposition 1 can be viewed as a particular case of Krishna (1989) we refer the reader to her paper for additional comments and turn now to the issue of quality selection.

4 Quality Selection: Imitation vs. Differentiation

As previously shown the sales quota alters equilibrium behavior in the pricing game thus equilibrium payoffs. We may expect firms' incentives to be altered at the quality stage as well. We show in this section that when quality is not costly, the presence of a capacity constraint fosters *minimal differentiation*. The intuition is easy to grasp. Given the incumbent's high quality and sales quota, the entrant can select the kind of equilibrium prevailing in the pricing game by adequately choosing its quality ($q \lesseqgtr \bar{q}(s_e)$ see Figure 3 below). In the laisser-faire regime the entrant incentives are to differentiate to $\frac{4}{7}$ (if possible) whereas in the binding regime the entrant's payoff does not depend on the incumbent price but on the residual demand and its own quality thus it is lead to choose the highest possible quality. It is clear that the binding regime will be prefered by the entrant only if the residual market is large enough or equivalently if the quota is not too loose ($q < q^*$ on Figure 3 below).

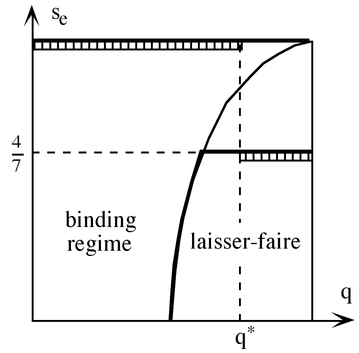

Figure 3

Proposition 2 *When quality is not costly, the entrant chooses the best quality (like the incumbent) if the incumbent's sales quota is lesser than $q^* = 71\%$ of the market size, otherwise the entrant differentiates to a ratio of $\frac{4}{7}$. The incumbent has an incentive to upgrade its own quality if it was not already the highest possible when anticipating entry.*

Proof When the price equilibrium is in mixed strategies the entrant's equilibrium profit can be computed with any of the prices in the support of its strategy. With the security price $p_e^s = s_e\frac{(1-q)}{2}$ the entrant has sales of $\frac{1-q}{2}$ thus its equilibrium profit is $\Pi_e(q, s_e) = \frac{s_e(1-q)^2}{4}$. For $q < \frac{1}{2}$ $(\leq \bar{q}(s_e))$ the price equilibrium is always in mixed strategies (binding regime on Figure 3 above) so that the highest quality (imitation of the incumbent) is the optimal choice. For $q > \frac{1}{2}$ the entrant can enter the binding regime by selecting a high quality s_e such that $q < \bar{q}(s_e)$ (beware of the axes reversal on Figure 3). The optimal choice is still imitation of the incumbent. Alternatively the entrant can choose the non-binding regime by selecting a low quality s_e such that $q > \bar{q}(s_e)$. In that case the laisser-faire equilibrium prevails and we have already seen in Lemma 1 that the unconditional best reply of the low quality firm was $\frac{4}{7}$. Thus the best differentiation decision is $\min\left\{\frac{4}{7}, \bar{q}^{-1}(q)\right\}$ (the kinked curve on Figure 3 above).

It remains to compare, for $q > \frac{1}{2}$, the respective benefits of imitation and differentiation. With imitation the profit is $\frac{(1-q)^2}{4}$ while under differentiation it is lesser or equal to $\Pi_e^*(\frac{4}{7}) = \frac{1}{48}$. The cut-off is $q^* = \frac{2\sqrt{3}-1}{2\sqrt{3}} \simeq 71\%$.

For the second part of the proposition, proved in Lemma 3 of the appendix, we start from $s_e < s_i < 1$ and show that in equilibrium of the binding regime $\Pi_i(q, s_i, s_e)$ is increasing with s_i. ■

This proposition is in striking contrast with the quite general principle of differentiation prevailing in standard differentiation models and we know of no other similar result in a purely non-cooperative setting of price competition within a single market[6] It is therefore important to assess the robustness of Proposition 2 with respect to the assumption of zero costs for quality.

To this end we assume that a firm has to incur a cost $\frac{s^2}{F}$ in order to produce the quality level s ($F > 0$). We maintain the assumption of an upper bound for quality at $s = 1$.[7] The cost is sunk when price competition takes place. Under laisser-faire (LF) and $F > 8$, the incumbent chooses maximal quality with $s_i^{LF} = 1$ while the entrant differentiate to $s_e^{LF} \simeq \frac{4}{7+\frac{100}{F}}$ leading to a final increasing concave payoff $\Pi_e^{LF}(F)$ (cf. Lemma 2 in the appendix). Under sales quota of q, the entrant's profit is $\frac{s_e(1-q)^2}{4} - \frac{s_e^2}{F}$ and is maximal for $s_e^{SQ} = \min\left\{1, \frac{F}{8}(1-q)^2\right\}$. Hence for $q = \frac{2}{3}$ and $F > 72$ the entrant chooses the highest quality in the binding regime. The final profit being $\frac{F-36}{36F} > \Pi_e^{LF}(F)$ the entrant has indeed no incentive to differentiate. More thorough computations[8] show that the results of Proposition 2 are robust to the introduction of the new assumptions in the following sense:

- If quality costs are “negligible“ (F large), product imitation prevails at the equilibrium of the entry game.
- If quality costs are “significant“, firms differentiate in equilibrium, but the degree of quality differentiation is lower than under laisser-faire.

Moreover, in the case where equilibrium differentiation prevails, both the low and the high quality levels are above those prevailing under laisser-faire. In other words the quality upgrading effect that follows the introduction of a sales quota is quite strong since all equilibrium quality selections improve upon the standard ones.

5 Sales Quota vs. Quality Minimum Standard

In this final section we aim at comparing quantitative restraints and MQS as instruments ensuring quality provision. We first consider the optimal sales quota for a government aiming at maximizing total welfare. On the one hand, any degree of differentiation leads to a lower consumer surplus thus the government should seek to maximize quality. On the other hand the sales quota tends to raises prices thereby limiting the access of poor consumers to the service. Trading-off these effects we obtain (proof in Lemma 4 of appendix)

Proposition 3 *When quality is not costly, the optimal sales quota to be imposed on the incumbent by the government is* 71% *of the market size.*

Setting the surplus under the Pareto optimum at 100%, *laisser-faire* yields a surplus of 91,6% while the optimal *sales quota* yields a surplus of 99,4%. To make a rough comparison with MQS we compute in Lemma 2 of the appendix the total surplus when the government imposes a MQS on the entrant. To reach the 99,4% level associated to the optimal sales quota the MQS should be set at $s \simeq 0.95$ which is 66% above the level chosen by the entrant under laisser-faire ($\frac{4}{7} \simeq 0.57$). The cost of enforcing such a dramatic increase of quality may well be prohibitive for the government.

Furthermore as the MQS is followed by a quasi Bertrand competition, equilibrium profits are very low (1% of the Pareto surplus for the entrant and 4% for the incumbent) thus as soon as there are some cost to acquire quality the entrant will not enter. As we show in the appendix a cost factor $F < 175$ would not permit the entrant to recoup the entry cost $\frac{s^2}{F}$ of producing the QMS $s \simeq 0.95$.[9]

Whatever the positive effect of a MQS, imposing a sales quota on top of it will lead to a quality increase. Consider indeed the optimal quality levels $\hat{s}_i$ and $\hat{s}_e$ retained by the firms under any MQS. By definition of optimality, it must be true that for each firm the corresponding level equalizes the marginal value of quality and its marginal costs, other things being equal. Consider then the effect of a sales quota imposed on a firm at a level corresponding to its equilibrium sales given $\hat{s}$. This raises both firms' profits by relaxing price competition (see Krishna (1989)) thus it also raises the marginal value of a quality improvement. Therefore, each firm would increase its quality. Intuitively, the main virtue of the capacity limitation, as compared to MQS, lies precisely in its ability to preserve a direct incentive to raise qualities, by ensuring large enough profits even if there is no quality differentiation.

To conclude let us recall the achievement of our model. Using a stylized vertical differentiation framework we have shown that a sales quota deeply alters firm's decisions

[6] Friedman & Thisse [93] obtain minimal differentiation in a horizontal differentiation framework but rely on partial price collusion. Schmitt [95] reports a minimal differentiation outcome but requires two distinct markets.

[7] We have also performed all our computations with an alternative cost function to ensure that our qualitative results still hold in the more realistic case where the top quality is infinitely costly to achieve.

[8] The formal derivation of these results have not been reported in the paper but are available upon request from the authors.

[9] For a positive cost of quality the upper limit q^* yielding imitation is slightly lesser than 71%.

regarding quality selection. When quality is not costly, the result is striking as duopolists provide the first-best quality level. A sales quota turns out to be a very effective mean of ensuring the provision of a high quality service as it relaxes price competition whatever the quality differential existing between firms. Roughly speaking, the constrained firm wishes to maximize revenues from selling at full capacity whereas the unconstrained one wishes to maximize the value of its residual market. In both cases, this is achieved through quality upgrading.

Appendix

Lemma 2 *Equilibrium under laisser-faire*

Proof Observe first from the set-up of the model that for $i \in \{l, h\}$, the consumer located at $x_i \equiv \frac{p_i}{s_i}$ enjoys zero utility. Hence every consumer with taste $x > x_i$ is willing to buy product i at the price p_i. Potential markets are respectively $[x_l; 1]$ and $[x_h; 1]$. We now identify the marginal consumer $\tilde{x}$ who is indifferent between the two products h and l. Solving for $\tilde{x}s_l - p_l = \tilde{x}s_h - p_h$, we obtain $\tilde{x}(p_l, p_h) = \frac{p_h - p_l}{s_h - s_l}$. Any consumer $x > \tilde{x}$ prefers h to l whereas the contrary prevails for $x < \tilde{x}$. Observing that quality levels can be re-scaled, we set $s_h = 1$ without loss of generality so that the demands are $D_l(p_l, p_h) = \begin{cases} \tilde{x} - x_l & \text{if } p_l \leq p_h s_l \\ 0 & \text{if } p_l > p_h s_l \end{cases}$ and $D_h(p_l, p_h) = \begin{cases} 1 - \tilde{x} & \text{if } p_l \leq p_h s_l \\ 1 - x_h & \text{if } p_l > p_h s_l \end{cases}$.

The particular shape of demands reflects the fact that in vertically differentiated markets the high quality firm may exclude the low quality one from the market. The latter, in order to enjoy a positive market share, must quote a price p_l significantly lower than p_h to compensate for its lower quality. Note also that since $x \in [0; 1]$, the market cannot be covered in equilibrium, expect perhaps for the case where $s_l = 0$.[10]

The profit function of the low quality firm l is $\Pi_l(p_h, p_l) = p_l D_l(p_h, p_l) = p_l(1 - s_l)\frac{p_h s_l - p_l}{s_l}$. The solution to $\frac{\partial \Pi_l}{\partial p_l} = 0$ is $\psi_l(p_h) \equiv \frac{p_h s_l}{2}$ and since $\psi_l(.)$ always lies strictly in the region where firm l enjoys a positive market share, the low quality best reply function is $\phi_l(p_h) = \psi_l(p_h)$.

In the monopoly region ($p_l > p_h s_l$), the best reply of the high quality firm is the monopoly price $\frac{1}{2}$ which is feasible if and only if $p_l > \frac{s_l}{2}$. Otherwise Π_h is strictly increasing in the monopoly region and we always reach the duopoly region where the

[10] As will be discussed later on, this particular assumption of partial market coverage does not affect qualitatively our results.

profit is $\Pi_h(p_h, p_l) = p_h D_h(p_h, p_l) = p_h \left[1 - \frac{p_h - p_l}{1 - s_l}\right]$. The solution to $\frac{\partial \Pi_h}{\partial p_h} = 0$ is $\psi_h(p_l) \equiv \frac{p_l + 1 - s_l}{2}$; it is interior to the monopoly area if $\psi_h(p_l) \leq \frac{p_l}{s_l}$ which holds true if and only if $p_l \leq \frac{s_l(1-s_l)}{2-s_l}$. Otherwise, $\Pi_h(., p_l)$ is strictly decreasing in the duopoly region and the frontier price $\frac{p_l}{s_l}$ is optimal. As we have $\frac{s_l(1-s_l)}{2-s_l} < \frac{s_l}{2}$, the (kinked) best reply of firm h is

$$\phi_h(p_l) = \begin{cases} \psi_h(p_l) & \text{if} \quad p_l \leq \frac{s_l(1-s_l)}{2-s_l} \\ \frac{p_l}{s_l} & \text{if} \quad \frac{s_l(1-s_l)}{2-s_l} \leq p_l \leq \frac{s_l}{2} \\ \frac{1}{2} & \text{if} \quad \frac{s_l}{2} \leq p_l \end{cases} \tag{5}$$

As one can see on Figure 1 in the text, the unlimited capacity equilibrium $(p_l^*, p_h^*) = \left(\frac{s_l(s_h - s_l)}{4s_h - s_l}, \frac{2s_h(s_h - s_l)}{4s_h - s_l}\right)$ is given by the intersection of ψ_l and ψ_h.

In the quality stage the profit of firm i is

$$\Pi_i(s_i, s_j) = \begin{cases} \Pi_l(s_j, s_i) - s_i^2/F & \text{if } s_i < s_j \\ \Pi_h(s_i, s_j) - s_i^2/F & \text{if } s_i > s_j \end{cases} \tag{6}$$

where $\Pi_h(s_h, s_l) \equiv p_h^* D_h^* = \frac{4s_h^2(s_h - s_l)}{(4s_h - s_l)^2}$ and $\Pi_l(s_h, s_l) \equiv p_l^* D_l^* = \frac{s_l s_h (s_h - s_l)}{(4s_h - s_l)^2}$. Let $s_h^*(F, s_l)$ solve $\frac{\partial \Pi_h}{\partial s_h} = \frac{2}{F}s_h$ and $s_l^*(F, s_h)$ solve $\frac{\partial \Pi_l}{\partial s_l} = \frac{2}{F}s_l$. The equilibrium is the solution of $s_i^* = s_h^*(F, s_e^*)$ and $s_e^* = s_l^*(F, s_i^*)$ because the entrant cannot leapfrog above the incumbent without making losses $\Pi_h(s_e, s_i^*) < \frac{s_e^2}{F}$ for any $s_e > s_i^*$. Our numerical computations (formulas are available upon request) show that for $F < 8$, $s_i^* \simeq \frac{F}{8}$ and $s_e^* \simeq \frac{F}{42}$ thus whenever the incumbent has not chosen the maximal quality the entrant differentiates at a very low ratio of $\frac{8}{42} \simeq 0.19$. If the cost for quality is lower ($F > 8$) then $s_i^* = 1$ and $s_e^* \simeq \frac{4}{7 + \frac{100}{F}} \underset{F \to +\infty}{\longrightarrow} 0.57$ and $\Pi_e^*(F) \simeq \frac{F\left(147F^3 + 6796F^2 + 93200F + 360000\right)}{16(7F+100)^2(3F+50)^2} \underset{F \to +\infty}{\longrightarrow} \frac{1}{48}$.

Let us consider the imposition of a MQS $z > \frac{4}{7}$ when quality is not costly. The entrant will choose a quality equal to the MQS as it wishes to differentiate more. The price equilibrium is $p_e^z = \frac{z(1-z)}{4-z}$, $p_i^z = \frac{2(1-z)}{4-z}$, equilibrium demands are $D_e^z = \frac{1}{4-z}$, $D_i^z = \frac{2}{4-z}$ and equilibrium profits are $\Pi_e^z = \frac{z(1-z)}{(4-z)^2}, \Pi_i^z = \frac{4(1-z)}{(4-z)^2}$ leading to a total surplus of

$$W^z = \int_{1-D_i^z}^{1} (x - p_i^z)dx + \int_{1-D_e^z - D_i^z}^{1-D_i^z} (zx - p_e^z)\,dx + \Pi_i^z + \Pi_e^z = \frac{12 - z - 2z^2}{2(4-z)^2} \tag{7}$$

an increasing concave almost linear function with limit $\frac{1}{2}$ at $z = 1$. To reach the level of surplus generated by the optimal sales quota $W(q^*) = 0.497$ the MQS must be set at $z = 0.95$.

The maximal QMS permitting entry for cost factor F solves $\frac{z(1-z)}{(4-z)^2} = \frac{z^2}{F}$ and is $z_F = \frac{\rho_F}{6} - \frac{6F - 32}{3\rho_F} + \frac{8}{3}$ where $\rho_F \equiv \sqrt[3]{12\sqrt{3F\left(4F^2 + 11F + 768\right)} - 180F - 512}$. The minimal cost factor permitting to implement $z = 0.95$ is $F \simeq 175$. ■

Lemma 3 *The Incumbent Quality Best Reply*

Proof In the binding regime, Proposition 1 tells us that the unique equilibrium sees the incumbent playing the pure strategy $\mu(q, s_e)$ while the entrant is playing the mixed strategy F_e defined as "play p_e^s and $\psi_e(\mu(q, s_e)) = \mu(q, s_d)\frac{s_d}{2}$ with respective probabilities α and $1-\alpha$.". Profits are therefore

$$\Pi_e(s_e) = p_e^s\left(1 - q - \frac{p_e^s}{s_e}\right) = \frac{s_e(1-q)^2}{4} \tag{8}$$

and

$$\Pi_i = \mu(q, s_e)\left[\alpha q + (1-\alpha)\left(1 - \tilde{x}\left(\mu(q, s_e), \mu(q, s_e)\frac{s_e}{2}\right)\right)\right] \tag{9}$$

We saw in the text that the entrant best reply is $BR_e(s_i/s_e \le s_i) = \begin{cases} s_i & \text{if } q \le 0.71 \\ \frac{4}{7}s_i & \text{if } q > 0.71 \end{cases}$.

We now want to show that the incumbent finds it optimal to choose the best available quality. To this end we need to compute precisely the mixed strategy equilibrium. We let s_e stands for $\frac{s_e}{s_i}$; it varies in $[0; 1]$.

The optimal incumbent price solves $\frac{\partial \Pi_i}{\partial p_i} = 0 \Leftrightarrow p_i = \frac{2(1-s_e)(1+\alpha q - 2\alpha) + (1-\alpha)s_e\mu(q,s_e)}{4(1-\alpha)}$. In equilibrium we also have $p_i = \mu(q, s_e)$ thus $\alpha = \frac{2(1-s_e)+\mu(q,s_e)(s_e-4)}{2(1-s_e)(1-q)+\mu(q,s_e)(s_e-4)}$. Plugging this last result into the profit function, we obtain $\Pi_i = \frac{2q\mu(q,s_e)^2}{2(1-s_e)(1-q)+\mu(q,s_e)(s_e-4)}$. Using $\mu(q, s_e) = (1-q)\sqrt{1-s_e}$, we finally express the reduced form of the incumbent's profit as $\Pi_i(q, s_e) \equiv \frac{2q(1-q)(1-s_e)}{(4-s_e)\sqrt{1-s_e}-2(1-s_e)}$. Algebraic manipulations show that is proportional to $s_e + s_e^2 - 2$ which is negative as $s_e < 1$. Recalling now that s_e stands for $\frac{s_e}{s_i}$, we can conclude that, as in laisser-faire, the incumbent's profit increases with its own quality, we have $BR_i(s_e/s_e \le s_i) = 1$. ■

Lemma 4 *The Optimal Sales Quota is* $q^* = 0.71$

Proof For $q \le 0.71$ the entrant chooses the highest quality and competition takes places in a market for an homogeneous good. This setting has been studied by Levitan & Shubik (1972) who identify the price equilibrium. Letting $\lambda(q) \equiv \frac{1-\sqrt{q(2-q)}}{2}$, firms play a mixed strategy with support $\left[\lambda(q); \frac{1-q}{2}\right]$ and cumulative distributions $F_e(p) = 1 - \frac{\lambda(q)}{p}$, $F_i(p) = \frac{p(1-p)-\lambda(q)(\lambda(q)-1)}{pq}$. Observe that $F_i(\lambda(q)) = 0$, $F_i\left(\frac{1-q}{2}\right) = 1$, $F_e(\lambda(q)) = 0$ and $F_e\left(\frac{1-q}{2}\right) < 1$ thus only the entrant has an atom at the upper price $\frac{1-q}{2}$. In this equilibrium the incumbent profit is $\Pi_i(q) = q\lambda(q)$ (at the lowest price it gets the whole

demand $1-\lambda(q)$ thus sells q because $\lambda(q) < \frac{1-q}{2} < 1-q$ implies that the incumbent is capacity constrained) while the entrant gets $\Pi_e(q) = \frac{(1-q)^2}{4}$ (at the highest price it gets the residual demand $1-q$). There is continuity for the entrant profit since $\Pi_e(q) = \lim_{s_e \to 1} \Pi_e(q, s_e)$ thus the exact imitation with $s_e = s_i = 1$ is indeed the optimal decision for the entrant.

We now compute the consumer surplus. The surplus of the consumer located at x is best understood by separating 2 cases:

- if $x > 1-q$ then $x > p_e$ because $p_e \le \frac{1-q}{2}$. The incumbent price p_i is the lowest with probability $F_i(p_e)$ in which case the consumer buys at the price p_i (because $x > p_e > p_i$ and the incumbent is not constrained) so that we need to compute an expectation. With complementary probability, the consumer buys at the entrant, thus the surplus of consumer x is

$$H(x, p_e) \equiv (x - p_e)(1 - F_i(p_e)) + \int_{\lambda(q)}^{p_e} (x - p_i)dF_i(p_i)$$

- if $x < 1-q$ and $x < p_e$ then the consumer is rationed by the incumbent and does not buy at all. When $x > p_e$ (and $x < 1-q$) the surplus of consumer x is $x - p_e$ because he is rationed by the incumbent.

Integrating with respect to the distribution of domestic prices, we have three cases according to the respective positions of x and the upper price limit:

- if $x < \frac{1-q}{2}$, $\underline{\underline{W}}(q, x) \equiv \int_{\lambda(q)}^{x} (x - p_e)dF_e(p_e)$
- if $\frac{1-q}{2} < x < 1-q$, $\underline{W}(q, x) \equiv \int_{\lambda(q)}^{\frac{1-q}{2}} (x - p_e)dF_e(p_e) + \left(x - \frac{1-q}{2}\right)\left(1 - F_e\left(\frac{1-q}{2}\right)\right)$
- if $1-q < x$, $\overline{W}(q, x) \equiv \int_{\lambda(q)}^{\frac{1-q}{2}} H(x, p_e)dF_e(p_e) + H\left(x, \frac{1-q}{2}\right)\left(1 - F_e\left(\frac{1-q}{2}\right)\right)$

Integrating with respect to the uniform distribution of consumers over the range of potential buyers i.e., $x \ge \lambda(q)$, we get the consumer surplus

$$W_C(q) \equiv \int_{\lambda(q)}^{\frac{1-q}{2}} \underline{\underline{W}}(q, x)dx + \int_{\frac{1-q}{2}}^{1-q} \underline{W}(q, x)dx + \int_{1-q}^{1} \overline{W}(q, x)dx = \frac{1 - 3q^2 + 4q + 2q\sqrt{q(2-q)}}{8} \tag{10}$$

which is an increasing concave function. Observe that $W_C(1) = \frac{1}{2}$ is the total surplus at the outcome of Bertrand competition between two identical products where no consumers refrain from buying, all consumers buy the best available quality and firms capture no rent. This is also the Pareto optimum in our simple model. To conclude the total surplus is $W(q) = C(q) + \Pi_i(q) + \Pi_e(q) = \frac{3-q^2+4q-2q\sqrt{q(2-q)}}{8}$; this function is increasing concave with $W(1) = \frac{1}{2}$ and $W(q^*) \simeq 0.497$.

If the sales quota is looser than $q^* \simeq 71\%$, the entrant will optimally differentiate to 4/7 and in the pricing sub-game, firms play the classical pure strategy equilibrium $\left(p_e^* = \frac{1}{14}, p_i^* = \frac{1}{4}\right)$. The optimal demands are thus $\left(D_e^* = \frac{7}{24}, D_i^* = \frac{7}{12}\right)$ and the profits $\left(\Pi_e^* = \frac{1}{48}, \Pi_i^* = \frac{7}{48}\right)$. The total surplus is easily computed as

$$W^* = \int_{1-D_i^*}^{1} (x - p_i^*)dx \;+\; \int_{1-D_e^*-D_i^*}^{1-D_i^*} \left(\frac{4}{7}x - p_e^*\right) dx \;+\; \Pi_i^* \;+\; \Pi_e^* = \frac{11}{24} \simeq 0.458 \quad (11)$$

hence the optimal choice for the public authority is to set the sales quota $q^* \simeq 71\%$ of the market.[11] ■

[11] The proof can be adapted to the case of convex quality cost with the same qualitative results.

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