

### **Abstract**

We compare the effect of a trade quota on products' quality selection under Cournot and Bertrand competition. In contrast with the Cournot case, a quota never induces quality downgrading under Bertrand competition, moreover the domain of effective quotas is larger under Bertrand than under Cournot.

# Trade Quotas, Quality Selection and the Mode of Competition\*

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## 1. Introduction

Despite of a marked trend towards freer trade, quotas and VERs are still present in many industries. The "Market Access Sectoral and Trade Barriers" Database maintained by the European Commission provides ample evidence on this point.<sup>1</sup> As recently argued by Maggi and Rodriguez-Clare (2000), the political influence of domestic producers and/or importers may explain why governments are likely to prefer quotas to tariffs. As a matter of fact a vast theoretical literature already helps us to better assess the likely influence of such trade restraints. Still, some key issues need to be clarified, in particular regarding the relationship between quota effects and the mode of competition.

A pathbreaking paper in this respect is Krishna (1989). She shows that because of the quantitative restrictions they impose at the market competition stage, quotas have qualitatively different impact depending on whether firms compete in quantity or price. On the other hand, the recent Industrial Organization literature shows that market stage effects tend to spillover to earlier stages of the game where long term decision (such as the selection of products attributes) are made. In a trade context, we may for instance expect that quotas affect products' quality selection but also that their impact on equilibrium quality choice depends on the mode of oligopolistic competition.

Herguera, Kujal and Petrakis (2000) carefully study the impact of quotas on quality selection by the firms under Cournot competition. However, in view of Krishna (1989)'s findings, it is not obvious that the conclusions they reach to extend to a Bertrand competition framework. The present note precisely aims at

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<sup>1</sup>See the website <http://makacdb.eu.int/mkdb/mkdb.pl>

testing the robustness of Herguera, Kujal and Petrakis (2000) conclusions to the mode of competition.

To this end, we develop a very simple example. Suppose a domestic government wishes to offer protection to an emerging industry facing international competition. For simplicity, we assume that the foreign producer is already committed to its quality before the quota is enforced. This allows us to concentrate on the quality selection of the domestic firm only. In order to analyze the role of the competition mode, we consider in turn the optimal quality selection under Cournot and Bertrand competition in the last stage. In our framework, the prediction of Herguera, Kujal and Petrakis (2000) is that, as compared to the free trade equilibrium, the domestic firm is likely to select a higher quality in the presence of a restrictive quota. However, if the quota is set just above the foreign sales under free trade equilibrium it is effective in the sense that it will alter the quality choice of the domestic firm. More precisely, the domestic firm selects a lower quality, as compared to free trade.

Comparing quality selection under Bertrand and Cournot competition, we show that for a large domain of quota values, the impact of the quota on quality selection does not depend on the mode of competition. However, we also show that, as compared to Cournot, the range where the quota is effective is significantly larger under Bertrand competition. Moreover, under Bertrand, the quota never induces quality downgrading by the domestic firm, as compared to the free trade benchmark.

## 2. The Model

Consumers preferences are derived from Mussa and Rosen (1978). Consumer  $i$  exhibits a taste for quality  $\theta_i$  and derives an indirect utility  $\theta_i s - p$  when consuming a product of quality  $s$ , bought at price  $p$ . Not consuming yields a utility of 0. Consumers' types are uniformly distributed in the  $[0, 1]$  interval. The density is 1, and is taken as a measure of the domestic market size.

In order to produce a quality level  $s$ , a firm has to incur a sunk cost  $c(s) = \frac{s^2}{8}$ . Marginal cost is zero for simplicity.

The sequence of decisions is the following. Before the game starts, a foreign producer sells as a monopolist in the domestic market a product of quality  $s_f = 1$ .<sup>2</sup> We assume that the monopolist is committed to this quality level.<sup>3</sup> Then the government chooses a level for the quota from an interval  $[q^{\min}, 1]$  with  $q^{\min} >$

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<sup>2</sup>Note that  $s_f = 1$  is indeed the optimal choice of a monopolist under the cost assumption  $c(s) = \frac{s^2}{8}$

<sup>3</sup>Thus, we assume that switching to another quality is too costly for the monopolist. This assumption may reflect the fact that the monopolist is also active in other foreign markets and does not find it profitable to alter its quality solely for the domestic market subject to trade restrictions.

0.<sup>4</sup> Given the quota, the domestic firm selects its quality  $s_d$ , before both firms compete in the last stage of the game. We consider in turn Cournot and Bertrand competition. Notice that given the specification of the sunk cost, we may restrict the analysis of domestic quality selection to  $s_d < 1$ , i.e. quality leapfrogging by the domestic firm is not an issue in our setting.

### 3. Cournot Competition

We first solve the last stage of the game, and then we go backward to study quality selection. The analysis of the Cournot game is straightforward. Given qualities, inverse demands are given by

$$p_f = 1 - x_f - x_d s_d \quad (1)$$

$$p_d = (1 - x_f - x_d) s_d \quad (2)$$

Under Free Trade, the Cournot equilibrium is directly obtained as

$$x_d^c(s_d) = \frac{1}{4 - s_d}; x_f^c(s_d) = \frac{2 - s_d}{4 - s_d}. \quad (3)$$

Domestic profits at the Cournot equilibrium are given by  $\pi_d^c = \frac{s_d}{(4 - s_d)^2} - \frac{s_d^2}{8}$ .

Maximizing this function with respect to  $s_d$ , we obtain the optimal quality selection by the domestic producer. The only relevant real root to the first order condition is  $s_d^c \simeq .3626$ .<sup>5</sup> This defines the optimal quality selection under Free Trade. Demand addressed to the foreign firm at the SPE is computed as  $x_f^c(s_d^c) \simeq 0.4502$ .

Assume now that the government has set a quota at level  $q$ . If the quota is binding at the Cournot stage, we have  $x_f = q$ . Using (2), the market clearing price for the domestic product is defined as

$$p_d^q = (1 - q - x_d) s_d \quad (4)$$

It is then direct to show that at the "quota-constrained" Cournot equilibrium we have  $x_d^{cq} = \frac{1 - q}{2}$ . By maximizing  $\pi_d^{cq} = \frac{(1 - q)^2 s_d}{4} - \frac{s_d^2}{8}$  with respect to  $s_d$ , we immediately obtain  $s_d^{cq} = (1 - q)^2$ . Thus, the optimal quality selection is decreasing in the quota.

Now, we address the following question: in which range is the quota binding? Once qualities have been selected, it is well known that the quota is effective if only it is strictly below the level of foreign demand at the Cournot equilibrium. However, since the quota is committed to before domestic quality is selected, a

<sup>4</sup>The introduction of a lower bound for the quota is made to avoid technical problems in the derivation of pricing games. The importance of this assumption is discussed below.

<sup>5</sup>Note that the optimal value can be computed analytically. However, to avoid cumbersome expressions, we retain here, as well as for the results to come, numerical approximations.

quota set above the Free Trade benchmark  $x_f^c(s_d^c)$  may be effective through its effect on quality selection.

Notice first, using (3), that  $\frac{2-s_d^{cq}}{4-s_d^{cq}} > q$  whenever  $q > .4608$ .  $x_f^c(s_d^c) \simeq 0.4502$ . Thus, there exist quota levels, above the Free Trade benchmark for which the foreign firm is quota-constrained in a Cournot equilibrium. To enforce this equilibrium, the domestic firm must select a quality level below the free trade one. Recall indeed that  $s_d^{cq}$  is decreasing in the quota. The intuition is straightforward. Suppose the quota is slightly above the Free Trade benchmark. The domestic firm may benefit from the protection of the quota if only it inflates foreign sales at the Cournot equilibrium. This is achieved through quality downgrading.

In order to identify the range in which this strategy is optimal for the domestic producer, we compare  $\frac{s_d^c}{(4-s_d^c)^2} - \frac{(s_d^c)^2}{8}$  and  $\frac{(1-q)^2 s_d^{cq}}{4} - \frac{(s_d^{cq})^2}{8}$ . Solving for  $q$ , we obtain as the unique relevant real root  $q^c \simeq .4558 < .4608$  which therefore defines the critical value below which the quota is effective when committed to before domestic quality is selected. The next proposition characterizes the impact of a quota under Cournot competition.

**Proposition 3.1.** *If  $q < .4502$  (the Free Trade benchmark) the domestic firm selects a quality above the Free Trade Equilibrium level; if  $q \in [.4502, .4558]$ , the domestic firm selects a quality below the Free Trade Equilibrium level; If  $q > .4558$ , the Free Trade Equilibrium quality level obtains.*

Notice that  $(1-q^c)^2 = .2961 < .3626$ . In other words, when we reach the level where the quota starts to affect the optimal quality choice, the domestic quality jumps down.

At this step it is useful to distinguish two notions of "effectiveness" for a quota. In general, we claim that a quota is effective if it alters equilibrium outcomes. Given this definition, we define a "short-run effective" quotas as altering Cournot outcomes given qualities, and "long-run" effective quotas as altering equilibrium outcomes through their effect on quality selection. Using these definitions, we notice that under Cournot competition, quotas above the foreign sales, conditional on qualities, are not short-run effective. However, quotas slightly above the Free Trade subgame perfect equilibrium are long-run effective.

#### 4. Bertrand Competition

Under Bertrand competition, demands addressed to the firms are given by

$$x_f = 1 - \frac{p_f - p_d}{1 - s_d} \quad (5)$$

$$x_d = \frac{p_f s_d - p_d}{s_d(1 - s_d)} \quad (6)$$

Let us now define firms' best reply in the Free Trade pricing game:

$$f_f(p_d) = \frac{1 - s_d + p_d}{2} \quad (7)$$

$$f_d(p_f) = \frac{p_f s_d}{2} \quad (8)$$

Straightforward computations yield the unique Nash equilibrium:

$$p_d^b = \frac{s_d(1 - s_d)}{4 - s_d}; p_f^b = \frac{2(1 - s_d)}{4 - s_d}. \quad (9)$$

Using these values, we note that  $x_d^b = \frac{1}{4 - s_d}$ ,  $x_f^b = \frac{2}{4 - s_d}$  and  $\pi_d^b = \frac{s_d(1 - s_d)}{(4 - s_d)^2} - \frac{s_d^2}{8}$ . Solving the first order condition on  $\pi_d^b$  for  $s_d$ , we obtain the optimal quality selection of the domestic firm as  $s_d^b \simeq .1923$  and the associated demand for the foreign firm is  $x_f^b(s_d^b) = .5252$ .

Let us then consider the presence of a quota. As shown by Krishna (1989), the first key difference between Cournot and Bertrand competition is that, under price competition, quotas above Free Trade are (almost) always "short-run" effective. This is so because a quota deeply alters the structure of the pricing game itself. Therefore, we first have to study in details the implication of the quota on price competition itself before we turn to the analysis of quality selection.

The keypoint is the following: Under price competition, consumers may be rationed by the foreign producer if its demand exceeds the quota level. Rationed consumers may then report their purchase on the domestic firm, whose effective sales increase. These rationing spillovers destroy the global concavity of the domestic firm's payoff and thereby make the existence of a pure strategy equilibrium problematic. In other words, the presence of the quota induce Bertrand-Edgeworth competition at the market stage. The analysis of this problem has been carefully studied in Krishna (1989) and we apply her methodology to our particular framework.

Notice first that in the Mussa and Rosen model, and if the domestic firm sells the low quality, any consumer that is rationed by the high quality foreign firm prefers to report his purchase on the domestic product rather than to refrain from buying. Accordingly, whenever  $x_f(p_d, p_f) > q$ , the number of consumers who report their purchase on firm  $d$  is given by  $x_f(p_d, p_f) - q$ . It is then direct to show that the residual demand addressed to the domestic firm is

$$x_d^q(p_d) = 1 - q - \frac{p_d}{s_d}. \quad (10)$$

Using (5) we now define  $p_d^q$  as the price of the domestic firm such that the quota is exactly binding. Formally, we solve  $1 - \frac{p_f - p_d}{1 - s_d} = q$  to obtain

$$p_d^q(p_f) = (1 - q)(1 - s_d) + p_f. \quad (11)$$

Whenever  $p_d \leq p_d^q$ , the free trade analysis applies and firms' demand are given by (5) and (6). Whenever  $p_d \geq p_d^q$ , the quota is binding and firms demand are  $x_d^q(p_d)$  and  $q$  respectively. Comparing the derivatives of the domestic demand function using (5) and (10), we observe that the domestic firm's demand exhibits an outward kink at  $p_d = p_d^q$ . This kink destroys the concavity of its profits. Thus, the existence of a pure strategy equilibrium is not guaranteed.

We now characterize the nature of equilibrium in the pricing game. To this end we define firm  $f$ 's best reply in the quota game, which we denote  $\varphi_f$ . Whenever  $x_f(p_d, p_f) \leq q$ , i.e. whenever  $p_d \leq p_d^q$ , the quota is not binding. Accordingly the foreign firm is better off playing  $f_f(p_d)$  as defined by (7). Whenever  $p_d \geq p_d^q$ , the foreign firm is better off selling the quota at the highest price. Solving  $1 - \frac{p_f - p_d}{1 - s_d} = q$  we obtain

$$p_f^q(p_d) = (1 - q)(1 - s_d) + p_d \quad (12)$$

. Thus, we have:

$$\varphi_f(p_d) = \begin{cases} f_f(p_d) & \text{iff } p_d \leq p_d^q \\ p_f^q(p_d) & \text{iff } p_d \geq p_d^q \end{cases} \quad (13)$$

Notice that the best reply is kinked and continuous, reflecting the fact that the foreigner's profit is concave in own price.

Using (10) we now define by  $\pi_d^q(p_d) = p_d x_d^q(p_d)$  the profit of the domestic firm when it benefits from spillovers. Notice that  $\pi_d^q(p_d)$  exhibits a local maximum for  $p_d^s = \frac{1-q}{2} s_d$  with profits equal to  $\frac{(1-q)^2}{4} s_d$ . In the domain where the quota is not binding, the best reply is  $f_d(p_f)$  as defined by (8), yielding payoff  $\pi_d(p_f) = \frac{p_f^2 s_d}{4(1-s_d)}$ . Solving  $\frac{p_f^2 s_d}{4(1-s_d)} = \frac{(1-q)^2}{4} s_d$  for  $p_f$ , we define the critical value  $\widetilde{p}_f = (1 - q)\sqrt{1 - s_d}$  for which the domestic firm is indifferent between the two strategies. The best reply correspondence is therefore:

$$\varphi_d(p_f) = \begin{cases} p_d^s & \text{iff } p_f \leq \widetilde{p}_f \\ f_d(p_f) & \text{iff } p_f \geq \widetilde{p}_f \end{cases} \quad (14)$$

Combining (13) and (14), we observe that there is only one candidate for a pure strategy equilibrium: the Free Trade Equilibrium as defined by (9).<sup>6</sup> A necessary and sufficient condition for this candidate to be an equilibrium is that  $p_f^b > \widetilde{p}_f$ . Direct computations show that this condition is satisfied if and only if  $q > q^b(s_d) = 1 - \frac{2\sqrt{1-s_d}}{4-s_d}$ . When this condition is not satisfied, there exist no pure strategy equilibrium. The natural candidate for a mixed strategy equilibrium is then the following one: The domestic firm randomizes between  $p_d^s$  and  $f_d(\widetilde{p}_f)$  while choosing the weight to put on each pure strategy to ensure that  $\widetilde{p}_f$  is indeed

<sup>6</sup>It is indeed immediate to check using (12) that  $\widetilde{p}_f < p_f^q(p_d^s)$ , which is sufficient to rule out any pure strategy equilibrium candidate in the quota binding domain of prices. Therefore, the only remaining candidates must lie in free trade region. There is only one such candidate: the free trade equilibrium.

a best reply for the foreign firm against the mixture. We call this equilibrium the Krishna equilibrium.<sup>7</sup>

Direct computations show that  $q^b(s_d) > \frac{2}{4-s_d}$ , for all  $s_d$ . Accordingly, whatever the domestic quality level, there always exist quota values above the free trade equilibrium level such that the quota is short-run effective. In particular, evaluating at the Free Trade subgame perfect equilibrium, we have  $q^b(s_d^b) = .5279 > x_f^b(s_d^b) = .5252$ . Notice that this is never the case under Cournot competition. This contrast in the short-run effectiveness of a quota depending on the mode of competition is the essence of Krishna (1989)'s findings.

Let us assume for the moment that the Krishna equilibrium always exist when the Free Trade one does not and study the issue of quality selection. It is not necessary to compute the mixed strategy explicitly for our present purpose. Indeed, the keypoint here is to note that in this equilibrium, the domestic firm earns exactly  $\pi_d^s = \frac{(1-q)^2}{4} s_d$  in the Krishna equilibrium.<sup>8</sup> Notice that this is exactly equivalent to the Cournot equilibrium payoffs under a binding quota. Therefore, if the Krishna equilibrium is played at the price competition stage, optimal quality selection by the domestic firm is identical to the quality selection made under Cournot, i.e.  $s_d = (1-q)^2$ , yielding a payoff  $\frac{(1-q)^4}{8}$  when we take the sunk cost into account.

In order to define the domain in which the quota affects quality selection, consider the optimal quality selection under Free Trade:  $s_d^b \simeq .1923$ . As noted above, we need a large enough quota  $q > q^b(s_d^b) = .5279$  for the Free Trade equilibrium to exist in the pricing game. On the other hand, it is always possible for the domestic firm to enforce the Krishna equilibrium by choosing a high enough quality. In order to identify the optimal strategy, we compare the corresponding payoffs. Direct computations indicate that  $\frac{s_d^b(1-s_d^b)}{(4-s_d^b)^2} - \frac{s_d^{b2}}{8} < \frac{(1-q)^4}{8}$  whenever  $q < .5301 \simeq q^b$ , where  $q^b$  defines the critical value above which the quota is neither long-run nor short-run effective under Bertrand competition.

Our findings are summarized in the following proposition.

**Proposition 4.1.** *If  $q < .5252$  (the Free Trade benchmark), the quota induces the selection of a higher domestic quality; any  $q \in [.5252, .5301]$ , is effective and induce a higher domestic quality; If  $q > .5301$ , the quota is totally ineffective.*

Notice that  $(1 - q^b)^2 = .2208 > .1923$ . In other words, when we reach the critical level at which the quota starts affecting quality choice, domestic quality jumps up.

A second comment pertains to the existence of the Krishna equilibrium. Indeed, this equilibrium does not always exist. For this equilibrium to exist, the

<sup>7</sup>We refer the interested reader to Krishna (1989), Theorem 2, for the detailed construction of this equilibrium.

<sup>8</sup>In a mixed strategy, the equilibrium payoff can be computed at any of the firm's atom. Since the foreign firm faces a pure strategy, its equilibrium payoff, computed at  $p_d^s$  must be  $\frac{(1-q)^2}{4} s_d$ .

foreign demand must satisfy the non-negativity constraint  $x_f(f_d(\widetilde{p}_f), \widetilde{p}_f) > 0$ . Direct computations show that this is the case if only  $q > 1 - 2\sqrt{1 - s_d}$ . In other words, there exists a lower bound on the quota value, which depends positively on  $s_d$  below which the Krishna equilibrium does not exist. This condition imposes restrictions on the admissible values of the quota only if  $s_d > 3/4$ . Notice then that  $(1 - q)^2 > 3/4$  if only  $q < .1339$ . As a consequence, our analysis is fully compelling if we assume  $q^{\min} > .1339$ . Hence our initial assumption of a lower bound on the admissible values for the quota.<sup>9</sup>

## 5. Comparing Cournot and Bertrand

As shown by Krishna (1989), the impact of a quota at the market stage depends on the mode of competition. This result is best observed in our framework by noting that a quota set slightly above Free Trade is never short-run effective under Cournot while it is always short-run effective under Bertrand. However, a quota also have long-run implications. By comparing quality selection by the domestic producer, we may now assess the long-run effectiveness of quotas. In order to assess the dependence of quality selection to the mode of competition, it is best to refer Figure 1.

*insert figure 1 about here*

The bold line depicts the quality selection as a function of the quota value under Cournot. The dashed one depicts the quality selection under Bertrand. Notice first that  $q^b > q^c$ . Thus the range for an effective quota is larger under Bertrand than under Cournot. Second, note that for very restrictive quotas the quality selection is invariant to the mode of competition. Last, as compared to their respective Free Trade values, the optimal quality cannot decrease because of the quota under Bertrand competition while there exists a domain of quota values, above the Free Trade benchmark for which the quota induces quality downgrading. Notice finally that when the quota becomes tighter the degree of product differentiation decreases.

These results have been obtained under quite a restrictive framework. Assuming that the foreign producer does not alter its quality selection as a response to the quota considerably eases the analysis. However, it is our belief that they capture some basic implications of quotas on quality selection. In Boccard and Wauthy (2000), we consider a more general game where the domestic and the foreign firms are free to choose any quality level, and thus induce any quality

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<sup>9</sup>When the Krishna equilibrium does not exist, there exists fully mixed strategy equilibria involving a finite number of atoms. We have not been able to fully characterize them. However, Levitan and Shubik (1972) provide a characterization for the particular case where  $s_d = 1$ . For the relevant domain of quota values, the payoff of the domestic firm at  $s_d = 1$  dominates the Free Trade benchmark. Hence, in this domain, it must be the case that the equilibrium quality is larger than the Free Trade one.

ranking between the domestic and the foreign product after the quota is implemented. We reach qualitatively similar conclusions: incentives to quality selection under Bertrand competition are in line with those prevailing under Cournot whenever the quota is effective. The degree of product differentiation decreases when the quota becomes tighter. When it becomes long-run effective, a quota induces a marked quality upgrading under Bertrand.

## References

- [1] Boccard N. and X. Wauthy (2000), Quantity restrictions and endogenous quality choice under price competition, mimeo
- [2] Herguera I., P. Kujal and E. Petrakis (2000), Quantity restrictions and endogenous quality choice, *International Journal of Industrial Organization*, 18, 1259-1277
- [3] Krishna K (1989), Trade restrictions as facilitating practices, *Journal of International Economics*, 26, 251-270
- [4] Levitan R. and M. Shubik (1972), Price duopoly and capacity constraints, *International Economic Review*, 13, 111-122
- [5] Maggi G. and A. Rodriguez-Clare (2000), Import penetration and the politics of trade protection, *Journal of International Economics*, 51, 287-304
- [6] Mussa M; and S. Rosen (1978), Monopoly and product quality, *Journal of Economic Theory*, 18, 301-317